Example Problems dealing with jet propulsion

These are intended to introduce some of the concepts needed for the course, and to let the student see some of the numbers involved, and their variations. The calculations involve some exponentials. The student is also asked to do repetitive calculations in order to plot functions: this is suited to spreadsheet applications.

Example 1

Calculate the density of air at a pressure of 1 atmosphere and a temperature of 25 deg. C.
Solution:
Pressure: 1 atmosphere = 101300 Newtons per square meter
Temperature: 25 deg. Celsius = 273.15 + 25
= 298.15 deg. Kelvin
The thermal equation of state relates the pressure P, absolute temperature T, and density ρ of a gas:

$P = \rho RT$, where

$R = R_u / MW$,

R_u being the Universal Gas Constant (8314.3 in SI units), and MW the molecular weight of the gas. Air is composed of 79% Nitrogen, and 21% Oxygen. The molecular weight of Nitrogen (N_2) is 28, and that of Oxygen (O_2) is 32. Thus the mean molecular weight is
MW = 0.79 * 28 + 0.21*32 = 28.84
Thus, the gas constant for air, R = 8314.3 / 28.84 = 288.29 $mK^{-1}s^{-2}$

ρ = 101300 / (298.15 * 288.29) = **1.1785 kg/m^3.**

It is useful to remember that in SI units, atmospheric pressure at sea-level is approximately 100,000, temperature is 300, and density is 1.2.

Example 2

The ideal efficiency of a jet engine can be found by representing the processes in the engine as a "Brayton cycle". The cycle efficiency is related to the ratio of the highest pressure in the engine P_B to the lowest pressure P_A. Thus, if an engine cycle takes air at 0.3 atmospheres, and the highest pressure in the engine is 10 atmospheres, the ideal cycle efficiency is

$$\eta = 1 - (P_B / P_A)^{(1-\gamma)\gamma}$$

where g is the ratio of the specific heats at constant pressure and volume = 1.4 for diatomic gases such as air.

$$\eta = 1 - (10 / 0.3)^{-0.2857}$$

= 0.6328, or 63.28%.

This is a measure of how efficiently the heat put into the engine can possibly be converted to work, if there are no losses. Actual efficiency of course will be lower. This shows why it is desirable to have as high a "pressure ratio" as possible in the engine, and why compressors are needed. Modern engines have compressor pressure ratios as high as 40.

Problem for Spreadsheet Practice: Plot the variation of ideal cycle efficiency as a function of pressure ratio, for ratios ranging from 1 to 40.

Example 3
Find the temperature at the compressor exit if the compressor inlet temperature is 100 deg. C, and the compressor pressure ratio is 32.

Solution:
The change in temperature of air in the compressor is related to the change in pressure by the "isentropic relation"

$$T_B / T_A = (P_B / P_A)^{(\gamma-1)/\gamma}$$

$T_A = 273.15 + 100 = 373.15$ K

$P_B / P_A = 32$

Therefore, T_B = **1004 K**

Example 4
Plot the stagnation temperature at the nose of an aircraft, as a function of flight Mach number, from Mach 0 to 2.5, when the atmospheric temperature is 35 deg. C.

Sample solution:
When high-speed air is slowed down to stagnant conditions, its temperature will rise according to the relation

$T_0 / T = 1 + 0.5 (\gamma-1)M^2$ where M is the Mach number, and T_0 is the "stagnation temperature", the temperature reached when the Mach number is reduced to zero. γ is the ratio of specific heats at constant pressure and volume, and is equal to 1.4 for air at moderate temperatures.

Atmospheric temperature T = 273.15 + 35 = 308.15 K.

At Mach 2.5,
stagnation temperature T_0 = 308.15 * (1 + 0.2*6.25)
= **693.34 K.**

Problem 1:
A jet engine cycle is designed to have the following features:
ambient pressure: 1 atmosphere
ambient temperature: 300K
highest temperature: 2000K
exhaust temperature: 500K. Calculate the Brayton cycle efficiency, and the highest pressure in the engine.

Problem 2:
At a standard altitude of 15000 m, an ideal ramjet engine is flying at maximum thrust at Mach 2.5. The highest temperature in the engine is 2500K. The fuel is methane, CH_4. Find
(1) the thrust per unit mass flow rate of air
(2) the fuel-to-air ratio
(3) the specific fuel consumption
(4) the actual thrust, if the nozzle throat diameter is 0.3m.
(5) the Brayton cycle efficiency
(6) the propulsive efficiency
(7) the thermal efficiency
(8) the exhaust temperature

Problem 3:
The compressor of a turbojet engine has a pressure ratio of 28. The upstream stagnation temperature and pressure are 300K and 100,000N/m^2 respectively. The stagnation temperature at the compressor exit is 850K. Find the stagnation pressure at the exit, and the compressor efficiency.

(35)

Problem 4:
A turbojet engine is flying at Mach 1.8 so that there is 1 oblique shock and a normal shock at the inlet. There are some small losses in the diffuser after the shock. The compressor and turbine exchange work with the fluid with no losses. There is some stagnation pressure loss in the combustor. The afterburner is OFF, but causes some loss in stagnation pressure. The exhaust nozzle is underexpanded. Sketch the variation of
a) Stagnation pressure
b) Stagnation temperature
c) static pressure
as a function of distance through the engine, going from ahead of the normal shock to downstream of the nozzle exit. Mark on the sketch the various stations, and state what they are (e.g., compressor, burner, etc.).

Pay careful attention to the slopes and relative heights of the lines that you draw. I will.

Problem 5:. Answer in complete sentences and show formulae where appropriate:
a) How do you find the limiting flight Mach number of an ideal ramjet engine at a given altitude?
b) How do you calculate the limiting mass flow rate of a turbojet engine at a given operating condition at a specified altitude and Mach number, given the total thrust at takeoff?

Problem 6:
The processes occurring in an engine can be represented as follows: The working fluid (air), which is initially at a pressure of 10^5 N/m^2 and a temperature of 300K is compressed isentropically, so that its temperature increases by a factor of 3. Heat is then added at constant pressure until the temperature reaches 1200K. Work is then extracted from the fluid in an isentropic process until the pressure returns to 10^5 N/m^2.
a) Calculate the cycle efficiency.
b) Calculate the temperature at the end of the work extraction.

Problem 7:. A turbojet engine is flying at Mach 1.5 so that there is a normal shock at the inlet. The heat addition in the burner can be assumed to occur with no pressure losses. There are no losses in the diffuser **after** the shock. The compressor and turbine exchange work with the fluid with no losses. The exhaust nozzle is isentropic. Sketch the variation of
a) Stagnation pressure
b) Stagnation temperature
as a function of distance through the engine, going from ahead of the normal shock to downstream of the nozzle exit. Mark on the sketch the various stations, and state what they are (e.g., compressor, burner, etc.).

Problem 8:. A turbojet engine is designed to fly at Mach 0.5 at an altitude where the temperature is 270K and the pressure is 4 x 10^4 N/m2. The stagnation pressure of the exhaust is 12 x 10^4 N/m^2 and the stagnation temperature is 800K. Find the propulsive efficiency. Assume that the fuel/air ratio is very small compared to 1. Assume that the nozzle is fully expanded. The value of the gas constant for air may be taken as 287 in SI units, and the ratio of specific heats may be taken as 1.4.

Problem 9:. A jet engine takes in air at 300K and 1 atmosphere pressure, and compresses it to 11 atmospheres before adding heat at constant pressure. The highest temperature reached is 1500K. Find the Brayton cycle efficiency. If the nozzle exit pressure is the same as the ambient pressure, what is the exhaust temperature?

Problem 10: A ramjet engine operates at an altitude where the pressure is 10,000N/m^2 and the temperature is 250K. The flight Mach number is 2.5. There is a 10% stagnation pressure drop in the inlet and diffuser. At the beginning of the burner duct, the Mach number is 0.3. Find the highest temperature that can be reached in the constant-area burner without exceeding the choking limit. Neglect the fuel mass flow rate.

Problem 11: A 2-D multiple-ramp inlet has 2 oblique shocks and a normal shock, and is operated under ideal design conditions. The normal shock occurs at a Mach number of 1.4. The leading oblique shock is inclined at 35 deg. to the flow as shown. Find the flight Mach number.

Problem 12: A design decision is being made on a turboprop engine. Explain how to find the optimum value of the percentage of available enthalpy that should be used to drive the power turbine. Show this value on a diagram. What does this value depend on? Why?