Example of a finite wing calculation

Problem: Need to find lift and the ratio of lift to induced drag, for a straight tapered, twisted wing.

Flight condition:

flight speed 100 m/s
freestream density: 1.2 kg/m^3
freestream static temperature: 288K

Wing Geometry:

Root chord: 3m.
Span: 15m.

Taper ratio: 0.5
Twist (linear): 2 degrees washout.
Section lift curve slope: 6.0 per radian.
Section zero-lift angle of attack: -2 deg.
Geometric angle of attack at mid semi-span : 5 deg.

Solution:

Tip chord = 0.5 * root chord = 1.5m.
Wing area S = 2*(3+1.5)*7.5/2 = 33.75m^2.

Aspect ratio = b^2/S = 225/33.75= 6.667

We'll use a symmetric, 2-coefficient Fourier series, using only odd coefficients. Thus, our assumed distribution of bound circulation along the span is given by:

$$\Gamma = 2bU_{\infty}[A_1 Sin\,\theta + A_3 Sin\,3\theta]$$

Pick 2 points on the wing, that are not the tip or the middle (these points already satisfy the equation)

The transformation from x-y coordinates to 'theta' coordinates is

$$y = \frac{b}{2} Cos\,\theta$$

Pick θ of $\pi/3$ and $\pi/6$ as locations where we will write down the equation. This is equivalent to satisfying the equation at y =3.75m and 6.495m respectively.

At $\theta = \pi/6$, the absolute angle of attack (the LHS of the Prandtl lifting line equation, equal to the local section angle of attack, relative to its zero-lift angle):

$$\alpha_a = 5 + 2 - \frac{1(6.495 - 3.75)}{7.5 - 3.75} = 6.2679^\circ = 0.1094 radians$$

chord $c(y) = 3 + \frac{6.495}{7.5}(1.5 - 3) = 1.701m$

So, writing out Prandtl's lifting line equation,

$$0.109396 = \frac{(4)(15)}{(6.0)(1.701)}\left[A_1 Sin\frac{\pi}{6} + A_3 Sin 3\frac{\pi}{6}\right] + \frac{A_1 Sin\frac{\pi}{6}}{Sin\frac{\pi}{6}} + 3\frac{A_3 Sin 3\frac{\pi}{6}}{Sin\frac{\pi}{6}}$$

$$0.109396 = 5.8789\left[\frac{A_1}{2} + A_3\right] + A_1 + 6A_3$$

so that

$0.109396 = 3.93945 A_1 + 11.8789 A_3$ ………………….Eq. (1)

Similarly, we write the equation at $\theta = \pi/6$. Note that here the chord is 2.25m, and the absolute angle of attack is 0.12217 radians.

$0.12217 = 4.849 A_1 + 0 A_3$ ………………….Eq. (2)

Solve these two equations. We get

$A_1 = 0.02519$
$A_3 = 0.0008538$

The lift and induced drag coefficients can be found as follows:

$$C_L = A_1 \pi (AR)$$
$$C_L = 0.5276$$

Spanwise efficiency factor

$$e = \frac{1}{1 + 3\left(\frac{A_3}{A_1}\right)^2} = 0.9966$$

$$C_{Di} = \frac{{C_L}^2}{\pi(AR)e} = 0.0133365$$

Lift –to-drag is simply the ratio of the lift coefficient to the induced drag coefficient, and comes out to be 39.56.