Concept: High Speed Aerodynamics

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Keywords: Mach nmber, compressible, subsonic, supersonic, transonic, shock, Prandtl-Meyer expansion

1. Definition

High speed aerodynamics spans the regime of aerodynamics where the flow speed can exceed 30 percent of the speed of sound.

2. Introduction

When the flow speed is above 30 percent of the speed of sound, the density variation from stagnation conditions due to speed changes exceeds 5 percent. Thus unlike the low speed, incompressible regime, velocity and pressure changes are coupled through the density changes in this compressible regime. Thermodynamics must be considered to understand the relationship between flow properties. The discussion in this essay builds on the concepts discussed in the Low Speed Aerodynamics essay. In addition to the circulation concept used to explain lift generation in low speed aerodynamics, the Kutta-Jowkoswky theorem shows that the effect of density differences offers the possibility of lift due to compressibility. The complexity of coupled property variations in the compressible flow regime is alleviated by using Mach number, the ratio of flow speed to the speed of sound, as the primary independent variable. The speed of sound is defined as the speed of propagation of an infinitesimal wave through the medium. It is also given by the differential change in pressure associated with density change in an isentropic process. Even when the flow may be assumed irrotational so that a velocity potential can be defined, the potential equation for compressible flow is nonlinear. It can be linearized in piecewise fashion, into linear equations valid in the subsonic and supersonic regimes respectively, under the assumption of small disturbances. The subsonic regime extends to the highest freestream Mach number where flow speed remains completely below the speed of sound everywhere around the object. Beyond this critical Mach number, supersonic flow regions appear somewhere over the object. This regime of mixed subsonic and supersonic flow is called the transonic regime. A stretching of spatial coordinates along the streamwise direction by a parameter involving the freestream Mach number reduces the compressible form of the potential equation to the Laplace equation. This is similar in form and principle to the Lorentz transformation used at relativistic speeds. The variation of aerodynamic properties in the subsonic regime can be computed to a good approximation using simple transformations in terms of Mach number from the results for low speed aerodynamics. The Prandtl-Glauert transformation relates coefficients of pressure, lift and pitching moment in compressible flow to their incompressible flow counterparts for a given thin object geometry. A different form of the transformation relates the changes in shape required to keep the pressure coefficient constant as Mach number increases. The Prandtl-Gothert transformation relates the aerodynamics of a wing in compressible flow, to the aerodynamics of a wing of lower aspect ratio, in incompressible flow. As Mach number increases, the center of pressure of an airfoil in subsonic flow moves downstream towards midchord. In general, as Mach number increases, the magnitude of the pressure coefficient at a point on the surface of a configuration increases. In other words, the lift curve slope of an airfoil increases with Mach number. Conversely, to maintain a given level of pressure coefficient, a thinner configuration suffices in compressible flow than in incompressible flow. Supersonic aerodynamics is completely different from subsonic aerodynamics, and cannot be derived by transformation from the subsonic. The linearized potential equation for supersonic flow has the form of a wave equation. The required boundary conditions are the upstream conditions, and the condition of flow tangency at the surface, and the far field boundary condition that disturbances cannot increase as distance from the object increases. The Mach waves form the characteristic directions in the solution of the wave equation, and define the upstream extent of the region where disturbances due to a given point are felt. Solutions to supersonic flow problems can be determined by marching downstream and along Mach waves from known upstream conditions. Simple results can be derived for the lift coefficient of a supersonic airfoil at angle of attack. The lift coefficient is given by 4 times the angle of attack in radians, divided by the square root of the quantity (square of freestream Mach number minus one). Thus the lift curve slope is highest near Mach 1, and decreases at higher Mach numbers. Camber has no effect on lift coefficient at supersonic speeds, but does have an effect on pitching moment. The center of pressure in supersonic flow is at midchord, for symmetric camber. A key difference in supersonic flow is that potential flow theory shows the occurrence of a wave drag. The difference from subsonic flow is explained thus. In subsonic flow, every point feels the effect of disturbances from every other point. Thus, as distance from the object increases, relative to the size of the object, the effects of disturbances from different parts of the object cancel each other out. In supersonic flow this does not always occur, as the some of the disturbances may proceed outwards along Mach waves parallel to each other, or even diverge. Thus the energy in these disturbances is presumed to be lost as propagating waves. In some regions, compression disturbances may merge into strong shocks, which incur a drop in stagnation pressure and are hence not isentropic. Shocks contribute heavily to drag. Expansion corners on the other hand produce Mach wave disturbances that diverge, and hence these flows are isentropic. For highly refined configurations, the shock strengths will be minimized, but a body of finite size always incurs wave drag in supersonic flow. Wave drag can be found by integrating the streamwise component of the normal force due to pressure on the surface of the object. For small disturbances, the wave drag coefficient can be decomposed as a linear sum of the contributions from angle of attack, camber and thickness. In each case, drag is proportional to the square of surface slope.

3. Advanced

Theodore von Karmans method of modeling disturbances using source distributions leads to results for the body shape for minimum wave drag under given constraints. The shape of the minimum-drag object with a given base area is expressed by the Karman Ogive shape, seen in the nose shapes of several supersonic fighter planes. The minimum drag shape for a given volume and length is described by the Sears-Haack body. This theory shows that the drag depends on the longitudinal distribution of the cross-sectional area of the body. Oswastitchs theory of equivalent body of revolution for far-field wave drag is a different perspective on the same finding, and permits the Sears-Haack result to be applied to wing-body-tail configurations. Another consequence of the above is the Transonic Area Rule that guides designers to minimize the maximum cross-sectional area of a vehicle that must go through the transonic regime. In transonic flow, zones of supersonic flow form, ending in shocks that cause large increases in drag. Sweeping (yawing) the wings is seen to mitigate the pressure distribution. Approximately, the same results are obtained as with a freestream Mach number equal to the component of the actual freestream perpendicular to the line joining center of pressure locations (i.e., the sweep line). Thus sweep is a common way of delaying the onset of critical Mach number and transonic drag divergence to Mach numbers much closer to 1. Hypersonic flow refers to a speed regime, usually well above Mach 5, where the product of Mach number and the slenderness ratio, called the Hypersonic Similarity Parameter, becomes of the order of one. In this regime, shock angles are very close to body surface slopes, so that there is thin shock layer close to the surface where the flow that goes through the shock is contained, and where the boundary layer where viscous heating and friction are felt, can merge with the region of vorticity behind the curving shock, into an Entropy Layer. Methods for analyzing hypersonic flow use these facts, but are discussed in a separate Concept Essay.

4. Supersets

Aerodynamics. Gas Dynamics. Thermodynamics.

5. Subsets

Subsonic transformations; slender wing theory; subsonic, supersonic, transonic and hypersonic aerodynamics

6. Other fields

Relativistic mass increase and Lorentz transformation. Hyperbolic and parabolic partial differential equations; wave equation.

7. Calculators/Applets

Hyperlinks are not given, because today Internet Search Engines and people have become quite adept and very quick at locating what they want, given some idea of what they should be seeking.

1. shock calculators
2. Prandtl-Meyer expansion calculators
3. Conical Flow calculators

8. Notes

High Speed Aerodynamics notes by N.M. Komerath

Supersonic Aerodynamics notes by Prof. Mason, VPT&SU

9. Byline

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10. References

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