Concept: Low Speed Aerodynamics

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1. Definition

When the possible range of variation of flow speed, from stagnation conditions to the maximum possible speed, is less than 30 percent of the speed of sound, the largest changes in density due to speed variations is less than 5 percent. This is the regime of low speed aerodynamics, otherwise known as incompressible flow aerodynamics. In this regime, density may be considered to be independent of flow speed. Accordingly, the flow velocity field may be calculated directly from the knowledge that mass is conserved. Variations of pressure may then be computed directly from the variations in flow speed, ignoring variations in density. The theory and experimental results developed in the incompressible flow regime form the basis for considering what happens at higher flow speeds, (compressible flow) where density changes do become significant.

2. Introduction

Low speed aerodynamics explains the generation of aerodynamic lift as being due to flow turning, causing a rate of change of the momentum of the flow, in a direction perpendicular to the free-stream or flight path. Thus lift is explained as the reaction by the flow, to the force that causes the flow to change direction. Drag is the loss of momentum of the flow along the freestream direction, as seen in the flow downstream of the object. The laws of physics are specialized to the case of a control volume through which fluid flows, to derive integral relations called the integral conservation equations, between the three dependent variables, density, velocity and pressure, and the independent variables of spatial dimensions and time. In the limit as the size of the control volume shrinks to zero, the differential conservation equations relate variations of the properties and their spatial and temporal gradients at a point. The conservation equations of mass and momentum (the latter derived from Newtons second law of motion relating forces and rate of change of momentum), along with the equations of state of the fluid medium (usually the Perfect Gas Laws) provide an adequate set of relations to analyze incompressible flow. Fluids may be assumed to be continua because the smallest volume of interest in low speed aerodynamics, which is on the order of a fraction of a cubic millimeter, still contains an enormous number of molecules, as may be verified using the Avogadro Number. Thus the motion of a given packet of fluid may be decomposed into four basic motions: translation of the

center of mass, rotation about any of three orthogonal axes passing through the center of mass, shear of surfaces of the packet relative to each other, causing a change of shape, and dilatation or volume expansion of the packet as the surfaces move along directions perpendicular to themselves. The vorticity vector is defined as twice the rotation vector. Where flow is incompressible, the dilatation is zero. Shear in itself causes no resistance in a fluid, but the resistance to the rate of shear strain causes stresses proportional to the viscosity of the fluid. A streamline can be defined in a steady two-dimensional flow field as the line such that the local flow velocity vector is everywhere tangent to the streamline. Thus the local inclination of the velocity vector may be directly obtained from the slope of the streamline. The Kutta-Joukowsky theorem, arising from Newtons second law of motion, shows that a lift force can arise from two reasons. The first is a turning of the flow, seen as the existence of vorticity, which is twice the rotation vector. The second is the force arising from compression of the flow in one region relative to another, seen in changes of dilatation of the flow. Where the flow is incompressible, dilatation is zero, so that lift can only arise from rotation. Thus lift on an object is proportional to the density, the freestream speed, and the circulation of the flow around the object. In vector terms, lift is proportional to the density and to the cross-product of the freestream velocity and the circulation vector. Where shear is negligible, as is usually the case away from flow boundaries (whether solid or free boundaries), the momentum conservation equation reduces to the Euler equation. Integrating along a streamline in steady incompressible flow gives the famous Bernoulli equation, equating the stagnation pressure to the sum of the static pressure and the dynamic pressure. The Pressure Coefficient is defined as the difference between the local pressure and the freestream static pressure, normalized by the freestream dynamic pressure. Where the rotation is negligible, the flow can be considered irrotational. With viscous dissipation of energy thus ruled out, the flow is conservative, and may thus be described as a potential flow. A scalar velocity potential may be defined in such a flow, such that the velocity vector is the spatial gradient of the potential. The differential equation of mass conservation (continuity equation) reduces for incompressible flow to the statement that dilatation is zero. Written in terms of the velocity potential, this becomes the Laplace equation, stating that the Laplacian operator applied to the potential is zero. The Laplace equation is linear, so that the solution to a complex field may be obtained as the linear combination of several elementary solutions. The solution to the Laplace equation for a specified set of boundary conditions gives the potential field, from which the velocity field may be found directly. The Bernoulli equation then gives the pressure field, from which the distribution of pressure coefficients over the surface of the object of interest may be found. Integrating this pressure coefficient distribution over the surface of the object gives the lift coefficient, and hence the lift force on the object. Four elementary solutions to the Laplace equation are the source, sink, doublet and vortex. Each of these contains a singularity, but the flow regime is defined as that excluding the singularity itself. The source and the sink describe mass flowing radially away and towards a point, respectively. The doublet is viewed as a source and sink of equal

strength, located in vanishingly close proximity but not concentricity. It is used to represent a force acting on the flow. A vortex induces turning of the flow around it, though all of the rotation is concentrated in the singularity at the center of the vortex, and the flow outside it is irrotational. Distributed singularities such as source, sink, doublet and vortex sheets, with spatially constant or varying strength distributions, form the basic building blocks of aerodynamic theory and analytical computations. The boundary conditions usually applied to describe the physical problem of flow around an object, are that the disturbances due to the object must not grow as distance from the object increases (far boundary condition), and that there cannot be any flow directed normal to the surface at the surface, or equivalently, that the velocity vector at the surface must be tangent to the surface (flow tangency surface boundary condition). The flow turning that is essential to lift generation cannot be calculated from the above set of boundary conditions. The correct amount of flow turning (circulation) is obtained by specifying that the flow must remain attached to the surface until the sharp trailing edge and separate smoothly from the trailing edge, so that the trailing edge is a stagnation line. This condition that approximates physical observation, enables calculation of a unique and correct amount of circulation, and is called the Kutta condition. Thus the actual process of calculating the flowfield around an object consists of the following steps: 1. Defining the geometry by locating numerous singularities of unknown strengths including vortex or doublet elements either along the surfaces, or along the camber lines of lifting objects. 2. Writing the surface boundary conditions specifying that the vector sum of the freestream velocity, and the velocity induced by all of the singularities, result in zero normal velocity at each of the control points along the surfaces. Along with the Kutta condition, this gives a set of N equations for the N unknown singularity strengths. 3. Solving this set of equations for the unknown singularity strengths. 4. Calculating the velocity at each control point, from the potential field which is given by the sum of all the potentials of the singularities and the freestream 5. Calculating the pressure coefficient distribution from the velocity distribution. 6. Calculating the lift by integrating the pressure coefficient distribution.

3. Advanced

Thin airfoil theory for incompressible flow uses the velocity difference induced by a vortex sheet to model the net shear and rotation of flow around a thin airfoil. The vortex sheet, of unknown strength distribution, is located along the camber line of the thin airfoil, and the strength distribution is calculated by solving the tangency boundary condition. The result shows that the vortex sheet strength reaches a singularity peak at the leading edge, falling off rapidly and then leveling off, reaching zero at the trailing edge per the Kutta condition. Thin airfoil theory gives the simple result that the lift curve slope of an ideal thin airfoil at small angle of attack, is 2 times Pi per radian. It also provides a simple analytical tool to calculate the lift and pitching moment due to both angle of attack and camber. The above discussions lead to the realization that

there are 3 basic ways of generating aerodynamic lift under incompressible flow conditions. The first is by directing the trailing edge (angle of attack). The second is by directing the trailing edge using camber. The third is by vortex-induced lift, where a vortex is located close to the suction surface, usually by flow rollup around highly swept, sharp leading edges. The rounded leading edge shape of an airfoil is thus seen to derive from the need to make the flow around the leading edge somewhat insensitive to angle of attack to keep it from separating, while the sharp trailing edge serves to precisely turn the flow. Camber allows flow turning even with zero angle of attack. Induced Drag: Incompressible (indeed, subsonic, sub-critical) potential flow around a 2-dimensional configuration such as an airfoil cannot generate drag. However, the generation of drag on a finite wing can be explained using potential flow. It is seen to be the result of vortices forming and trailing back from the wing tips, and all along the wing span. Fundamentally, the energy of spin of these vortices comes from the flow kinetic energy, and hence detracts from the momentum directed along the freestream. An elegant way of calculating this lift-induced drag and the loss of lift all along a wing due to the presence of the wing tips is to view the velocity field of the wing as being induced by the vortex system created by the flow around the wing. This system consists of the bound vortex or bound circulation, which is the vortex equivalent to the lift distribution along the wing, the trailing vortex sheet along the wing span, and the trailing tip vortices. To complete the system and conserve the total rotation of the flow at zero, one must add the starting vortex whose strength distribution is equal and opposite to that of the bound circulation distribution. This vortex system induces a downward velocity (assuming that the desired lift direction is upward) everywhere inside the region bounded by the tip vortices, the bound vortex and the starting vortex. The resultant effective freestream velocity is thus reduced in angle of attack, thus explaining a lift reduction compared to airfoil values. The lift vector, being perpendicular to the effective freestream, is thus tilted towards the downstream direction. The component perpendicular to the actual freestream is the lift, while the component along the actual freestream is the induced drag. Boundary layer theory, simplified from the full Navier-Stokes equations under the assumption of high Reynolds number, permits analytical calculation of the viscous effects at the surface, and the nature of the wake downstream of the object. This also allows calculation of the region where flow might separate from the surface due to the adverse pressure gradient. The actual variation of airfoil profile drag with angle of attack is thus explained.

4. Supersets

Aerodynamics.

5. Subsets

Lifting line theory, potential flow theory, thin airfoil theory, slender wing theory.

6. Other fields

hydrodynamics, sailing, windsurfing.

7. Notes

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8. Byline

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9. References

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