

Concept: Aircraft Performance

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1. Definition

When the possible range of variation of flow speed, from stagnation conditions to the maximum possible speed, is less than 30 percent of the speed of sound, the largest changes in density due to speed variations is less than 5 percent. This is the regime of low speed aerodynamics, otherwise known as incompressible flow aerodynamics. In this regime, density may be considered to be independent of flow speed. Accordingly, the flow velocity field may be calculated directly from the knowledge that mass is conserved. Variations of pressure may then be computed directly from the variations in flow speed, ignoring variations in density. The theory and experimental results developed in the incompressible flow regime form the basis for considering what happens at higher flow speeds, (compressible flow) where density changes do become significant.

2. Introduction

Under Performance, we analyze issues including: a) How long can the aircraft stay in the air (endurance)? b) How far can it fly (range)? c) How much payload can it carry on a given mission? d) How long will it take to reach altitude (climb performance)? e) What fast can it fly? f) How slowly can it fly? f) How long a runway does it need to take off? g) How long a runway does it need to land? h) How quickly can it turn, pitch, and roll? i) What is its minimum turning radius while flying? j) What are the boundaries of its flight envelope?

2.1. Steady Flight

Primary cases considered are those of steady level flight, steady climb or sink, coordinated turn at constant speed and altitude, takeoff and landing. In steady level flight, climb or descent, the lift and thrust balance gravity and drag, with no net acceleration in any direction. In practice, since density changes with altitude, steady climb or descent require some continuous adjustment of airplane attitude, thrust, control surface settings or a combination of these.

26 *2.2. Coordinated Turn*

27 In a coordinated turn, the aircraft rolls its lift vector so that the aerodynamic lift has a component
28 directed towards the center of curvature of the turn. This provides the centripetal acceleration, balancing
29 the inertial force as the aircraft describes a circular arc. The component of the lift along the vertical must
30 balance gravity, and the thrust must balance drag, if the turn is to be conducted at constant altitude. The
31 aircraft nose is pointed along the tangent to the curved path, to maintain the coordination of the turn,
32 usually by deflecting the rudder. If the turn is not perfectly coordinated, the aircraft will experience some
33 degree of sideslip. The rate of turn is seen to be limited by the thrust, the maximum lift coefficient, and by
34 the structural or human limits on load factor.

35 *2.3. Drag Polar*

36 The relation between lift and drag of an aircraft is usually of the form

$$C_D = C_{D0} + KC_L^2 \tag{1}$$

37 where C_D is the drag coefficient, C_L is the lift coefficient, the zero subscript denotes the value at zero lift,
38 and K is approximately constant. Further K is inversely related to the wing aspect ratio AR and is slightly
39 higher than the ideal value $1/(\pi AR)$ The drag polar of an aircraft is a chart of the relation

$$C_L = f(C_D) \tag{2}$$

40 It resembles a parabola.

41 *2.4. Steady Flight Envelope*

42 The steady flight envelope defines the conditions for maximum and minimum speeds at each given altitude.
43 The ceiling for a given speed and weight is defined as the altitude where the maximum rate of steady climb
44 falls below a specified minimum. Generally, the lower limit on speed is dictated by the stalling lift coefficient,
45 or in some cases by the thrust available. The highest speed is dictated either by the thrust, or by structural
46 load considerations.

47 *2.5. Speed for Minimum Drag*

48 Unlike other types of vehicles, aircraft operating on aerodynamic lift have a non-zero speed for minimum
49 drag. This is because sufficient lift must be produced to balance the weight. The lift-induced drag rises at
50 low speeds because higher lift coefficients are needed to maintain sufficient lift. At higher speeds, induced
51 drag is low, but the profile drag rises. Thus the speed for minimum drag is the speed where the lift-induced
52 drag and the profile drag are equal, each being half of the total drag. Higher wing aspect ratio helps to reduce

53 induced drag coefficients and thus push the speed for minimum drag lower, while streamlining to minimize
54 flow separation and turbulent skin friction, reduce the profile drag, and push the speed for minimum drag
55 higher.

56 *2.6. Range and Endurance*

57 The range of an aircraft is the distance flown before the fuel reserve falls below that required to maintain
58 sufficient margin of safety. The range is found by integrating the distance travelled per unit fuel expended
59 per unit thrust (this is the reciprocal of the thrust-specific fuel consumption), from the starting total weight
60 with full fuel, to the weight with only the minimum reserve of fuel left. As weight changes, the speed, altitude
61 or both may change, so there are multiple choices of flight profile. Similarly, the endurance is given by the
62 total amount of time for which the aircraft stays in straight and level flight before the fuel is exhausted. One
63 choice is to fly at maximum Lift/ Drag ratio, where the speed is held at the speed for minimum drag. This
64 would give best endurance. The speed for highest range is usually higher than that for maximum endurance.

65 *2.7. Takeoff and Landing Distances*

66 The takeoff roll distance for an aircraft is calculated from the net thrust available for acceleration,
67 accounting for ground roll friction and air drag. The distance travelled before the speed exceeds the liftoff
68 speed, where lift exceeds weight, is calculated for the given wing lift coefficient. The runway length needed
69 for takeoff is twice the takeoff roll length, since the aircraft must be able to brake to a halt in the remaining
70 distance, if takeoff is aborted at the liftoff speed. Multi-engined aircraft are generally required to be able
71 to take off, circle, dump fuel and return to land, with one engine failing at the liftoff point. The length
72 required for landing includes the distance over which the aircraft settles to the ground, flying nearly parallel
73 to the ground, before touchdown and deceleration. The procedure for landing is to aim the descent trajectory
74 towards a point just past the runway threshold, then rotate the aircraft, increasing lift coefficient continuously,
75 to level the aircraft off just above the runway to float in ground effect, and keep increasing angle of attack as
76 the speed decreases. One model of a perfect touchdown is where the aircraft stalls just as the main landing
77 gear touches down, so that there is no bounce.

78 *2.8. High Lift Devices*

79 In order to reduce the speed at landing and takeoff, devices such as flaps, slats and slots are used on
80 the wings. These increase the camber, delay flow separation by allowing high-pressure air from the lower
81 surface to energize the boundary layer on the upper surface, and increase the effective planform area of the
82 wings. The lift increase due to these devices, appears as a large increase in the effective lift coefficient of the
83 wing when referred to its original planform area. Thus lift coefficients higher than 3 are sometimes shown
84 for complicated sets of high lift devices used on large aircraft. The deployment of high-lift devices generally

85 comes at a high cost in drag. In the case of flaps, the highest values of flap deflection used in the final stage
86 of landing descent, serve to increase drag, more than they increase lift.

87 *2.9. Laminar Flow Performance*

88 At the other extreme of performance is the problem of keeping the zero-lift drag coefficient as low as
89 possible. For most aircraft cruise speeds, skin friction drag is a large contributor to total drag. Skin friction
90 increases sharply when the boundary layer transitions from laminar to turbulent state. Thus airplane
91 designers seek to maintain a laminar boundary layer as far downstream from the leading edge and nose as
92 possible during cruise. Usual approaches to this problem involve using suitably smooth surfaces with no
93 seams or fasteners protruding into the boundary layer, and minimizing the value of pressure gradients along
94 the flow direction to avoid regions of high adverse pressure gradient which would promote transition. One
95 ideal design state would be a configuration where the boundary layer remains laminar for the majority of
96 the wing and fuselage surface, and the rest of the surface is contoured to maintain a state very close to flow
97 separation. Flow separation is a condition where the skin friction goes to zero. Obviously this is an ideal
98 condition, because small perturbations might cause large fluctuations in the separation line.

99 *2.10. Spacecraft Performance*

100 In the case of space missions, the Rocket Equation attributed to Konstantin Tsiolkovsky relates the three
101 primary parameters. The delta V, expresses in units of velocity, such as meters per second, is actually a
102 measure of the square root of the energy per unit mass, required to change trajectory from one orbit to
103 another. Thus the delta V to go from the surface of the Earth, to an eastward circular orbit at about 400
104 kilometers above the launch site, is approximately 9 kilometers per second. The Specific Impulse, known
105 as Isp, is expressed in seconds, and is a measure of the speed difference imparted by the rocket engine
106 to the propellant, divided by the standard constant, 9.8 meters per second-squared. It is a metric of the
107 momentum added per unit mass of propellant expended. The rocket equation relates these to the Mass
108 Ratio of the vehicle, which is the initial mass before propellant ejection commenced, to the final mass when
109 enough propellant is ejected to achieve the desired delta v. The mass ratio is thus larger than one, and
110 usually much larger than one for a rocket going through a large delta v such as that needed to go into orbit
111 from the Earth's surface. The ideal rocket equation assumes that simply gaining tangential velocity with
112 respect to a gravitational center, such as the center of the Earth, is adequate for the inertia to balance the
113 centripetal force of gravity and achieve a stable orbit. In practice, the rocket equation also accounts for the
114 energy required to rise radially from Earth against gravity, and to overcome aerodynamic drag, or, in the
115 case of vehicles in space, the effects of the radiation pressure of sunlight and the gravitational pull of all
116 neighboring bodies.

117 *2.11. Hohman Transfer vs. Continuous Thrust*

118 Theoretically the most efficient way to impart kinetic energy to a vehicle is impulsive launch, expending
119 all the propellant instantly so that no energy is wasted lifting or accelerating propellant with the vehicle. Of
120 course this would destroy any vehicle other than a cannonball, so large rockets use gentle accelerations of no
121 more than 1.4 to 3 times the acceleration due to gravity. The advantage of impulsive thrust is used in the
122 Hohman transfer maneuver between different orbits in space. A rocket is launched into a highly eccentric
123 elliptical trajectory. At its highest point, more thrust is added quickly. This sends the vehicle into a circular
124 orbit at the desired height, or into a new orbit that takes it close to another heavenly body. Reaching the
125 same final orbit using continuous, gradual thrust would require roughly twice as much expenditure of energy.
126 However, continuous thrust is still an attractive option for long missions in space, because a small amount of
127 thrust can be generated using electric propulsion engines that accelerate propellant to extremely high speeds
128 compared to the chemical engines used for the initial ascent from Earth.

129 *2.12. Gravity Assist or Slingshot Maneuvers*

130 Yuri Kondratyuk suggested in 1918 that a spacecraft could use the gravitational attraction of the moons
131 of planets to accelerate and decelerate at the two ends of a journey between planets. The Luna 3 probe used
132 the gravity of the Moon in photographing the far side of the Moon in 1959. Michael Minovitch pointed out
133 that the gravitational pull of planets along the trajectory of a spacecraft could be used to accelerate the craft
134 towards other planets. The Mariner 10 probe used this gravitational slingshot maneuver around Venus to
135 reach Mercury at a speed small enough to go into orbit around Mercury. The Voyager missions used the rare
136 alignment of the outer planets to receive gravitational assists from Jupiter and Saturn to go on to Uranus
137 and Neptune, before doing another slingshot around Jupiter and Saturn to escape the solar system. Today
138 gravity assist is part of the mission planning for all exploration missions, and even for missions near Earth
139 where the gravity of the Moon is used.

140 *2.13. Aerobraking*

141 Another way of rapidly altering the momentum vector of a spacecraft, is to have it go through a brief
142 encounter with the atmosphere of a planet. This provides a strong braking force with respect to the planet.
143 Obviously it requires very precise orientation and height selection, and a suitable heat shield pointed in the
144 right direction.

145 **3. Supersets**

146 Dynamics, Aerodynamics, Propulsion Subsets: Range, Endurance, Speed for Minimum Drag, Ceiling,
147 Stall Speed, Flight Envelope, Load Factor.

148 **4. Other fields**

149 Rocket performance, Mass ratio and specific impulse

150 **5. Notes**

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152 **6. Byline**

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