Concept: VORTEX

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1. Definition

A vortex is a region in a fluid, where the flow rotates in an organized manner. Such regions are formed naturally when fluid experiences a favorable pressure gradient, i.e., a region of lower pressure is connected to a region where the pressure is higher. Examples abound. Water draining out of a bathtub or washbasin is something we see every day: it naturally develops a swirl as it is "sucked down" through the drain hole. Similar phenomena occur when air near the ground gets heated, and rises: the rising plume does not stay as one parallel set of streamlines, but develops a rotation. Sometimes the rotation can get extremely strong, as in a tornado. On a larger scale, the "eye" of a hurricane clearly shows a vortex. Getting a little more ambitious, we see the same pattern when we look at the arrangement of stars and dust in a galaxy, like our own Milky Way.

2. Introduction

Considering vortices occurring in aircraft applications, such as the tip vortex from a wing or rotor blade, note that the fluid elements inside the vortex do not necessarily remain inside the vortex: there can be continuous exchange of matter through a vortex. Hence the definition of a vortex as a region with certain characteristics, rather than as a distinct entity. Viewed in another sense, the effect of an airfoil on a freestream can be replaced by the effect of a vortex in the freestream: it induces a rotation in the flow.

If something is so prevalent in nature, there must be simple ways of describing the fundamental phenomenon. Consider the following explanation, based on conservation of angular momentum (this follows from Newton's Second Law of Motion): We allow water to collect in a washbasin, and let it sit for some time until it is more-or-less stagnant. Then we open the drain-hole by pushing on the knob which controls it, again a symmetric maneuver: this should affect the fluid all around the drain hole at the same instant, and to the same amount. The water starts flowing into the drain hole (since the pressure is lower at the hole). A "sink" flow develops, one where the flow is moving radially into the hole.

Where is the rotation? Well, here is one way by which it can start. Suppose there is a small disturbance in the flow at some radius ”r” at time ”t”, in this sink flow. A ”disturbance” in this context is something which makes the flow go a bit sideways, rather than straight along the radius. Now, if one calculates the angular momentum of all the fluid within the radius ”r”, one finds that most of it has zero angular momentum, but this small element has a tangential ”vt” at distance r. So the total angular momentum of the fluid within the radius ”r” is ”rvt”.

Newton takes over at this point. The angular momentum remains constant unless there is some torque to change it. Away from the solid surface of the washbasin, where there will be some viscous drag, there is no such torque. The vortex is formed. According to our physics so far, the variation of tangential velocity with radial distance is a hyperbolic relationship, as shown below.

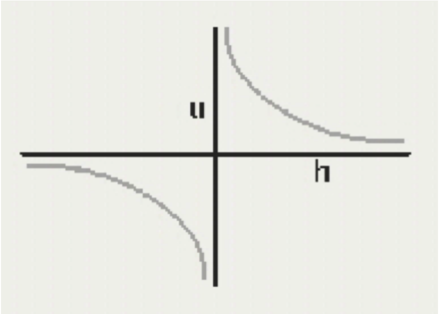

Figure 1: Ideal Vortex Velocity Profile

3. Structure

What happens at the center? The figure above predicts that tangential velocity should go to +infinity as you approach from one side, and -infinity if you approach from the other side. Obviously, at r=0, something drastic must happen because there must be infinite shear strain instead. In the case of a drain-hole, there is no water at the very center: its probably still air. So we don’t have to worry whether infinite velocity occurs at zero radius. In the case of a vortex in air, such as a tornado, we do have to worry about this ”core” region. Obviously the swirl velocity cannot become infinite at the center of the vortex. Instead, as we get closer to the center, and fluid encounters a sharp change in velocity in a small distance, viscous stress develops. The ”velocity discontinuity” gets smoothed out. In this region, the velocity changes smoothly from a high positive at one edge to a high negative at the other, going through zero. This is what happens, for example, if there were a solid cylinder placed in this core region: it would spin, so that its surface velocity matched the tangential velocity of the fluid at its outer radius. The velocity at the center of the spinning cylinder is zero. Thus, in the crudest model of a ”two-dimensional vortex”, a region of ”solid-body rotation” forms inside this core region. The vortex velocity profile now looks as shown below: A short time later, (time

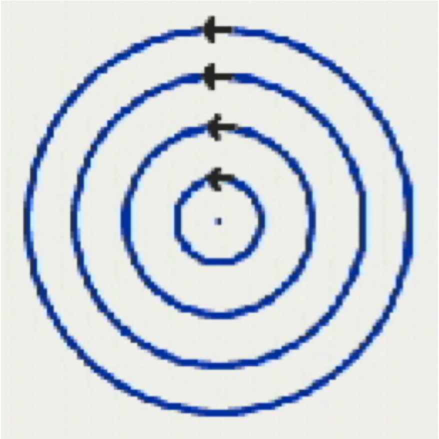

Figure 2: Irrotational Flow Outside the Vortex Core

t+dt), the fluid which was at radius r has moved inward by dr. A circle of radius "r-dr" will enclose the fluid containing the disturbance. Since angular momentum is conserved, the total angular momentum is still "—h—", except that it is now within a smaller radius. The tangential velocity of the fluid must now have increased, inversely proportional to r. The fluid swirls around faster and faster. The magnitude of the angular velocity, given by increases, inversely proportional to the square of the radius. Now we must point to a strange aspect. In fluid mechanics, we can go to the ideal 2-D point vortex, and plot streamlines of the fluid. They are all circles (if one ignores the sink, which is a 3-D phenomenon). Thus, we can say that in the ideal 2-D vortex, each fluid element goes traveling in a circle, revolving around the center. Imagine a micro-bus full of tourists going along one of these streamlines. The bus is so narrow that it stays on one streamline. The bus is not crossing the markers of this "streamline" so its not going into a region where the velocity is different (if it did, it would spin). The people sitting on the left side of the bus can see the vortex center always on their left; the people on the right always see the outside regions of the flow. The bus is not actually spinning. Thus, we can say that the flow outside the center of the vortex is "irrotational" . The fluid inside the core is, on the other hand, clearly "rotational". Thus the ideal vortex can be modeled as a region of irrotational fluid, going around a region of rotational fluid, called the core. In ideal fluid dynamics (i.e., "potential flow") we limit our flow domain to that outside the rotational region. Here the "influence" of the vortex in the ideal fluid can be described by the Biot-Savart Law:

4. Biot-Savart Law

4.1. The 3-D vortex, and a simple view of "vortex breakdown"

Now let's return to reality: in the "core" region of a real vortex, there is most certainly a strong "axial" flow: flow along the axis of the vortex. This is the "sink" effect, which produced the vortex in the first place.

What causes the sink effect is a matter of circumstance. In the case of a Galactic Black Hole (of which we can speak confidently, since no one who has been there is likely to return and contradict us) gravity provides the "sink" effect: it pulls everything towards it, never to give up anything until a Final Explosion, perhaps. In the case of the kitchen sink, it is again gravity, and the fact that the opening of the drain-hole opened a region of low pressure to which the fluid can flow. What if the drain-hole is stopped suddenly? The pressure gradient becomes less and less favorable, as the liquid level builds up in the drain hole. The axial flow stops. Snap! the rotation also stops. Why? This can be explained by a combination of conservation of mass, and conservation of momentum. When the axial flow stops, the mechanism for strengthening the rotation, described in the equations above, also stops: there is no radial flow of the liquid. In the case of a vortex over a delta wing, a similar phenomenon is observed. The pressure gradient becomes less favorable, and the flow along the axis, which was coming along a thin tubular region which we called the "core", slows down. Conservation of mass dictates that the diameter of the core must increase when the fluid slows down: to accommodate the same mass flow rate, the tube diameter must get bigger. Thus, the edge of the core, where the rotation is fastest, moves out to a larger radius, like an ice dancer stretching her arms. Conservation of angular momentum takes over: the rotation slows down. This is an extremely simplified explanation of the phenomenon of "vortex bursting" or "vortex breakdown", a topic on which debates rage, with the fervor of a religious war. We are not getting into the chicken-or-egg issue of what happens first: does the rotation slow down, forcing an increase in pressure, or vice versa? What was the first disturbance? You can read all the papers on these issues and decide for yourself.

4.2. Vortex Interactions

In a real fluid, a vortex might sustain itself for a very long period, because, away from the core, there is very little viscous stress. Thus, the tip vortex left behind by an aircraft might persist in the atmosphere for quite a long time (several minutes). A hurricane might persist for days. When this organized fluid movement encounters other regions (say a solid surface, or another vortex), regions of high shear strain develop, and viscous stresses develop. This might have several kinds of effects.

4.3. 3.3 The "Image Vortex" model

We can look at vortex interactions by considering the effect of each vortex at the center of the other. We can consider each vortex to be located in the infinite region of influence of the other vortex, and use the Biot-Savart Law to calculate the "velocity induced at the center of vortex A by the vortex B" for example. This tells us how each vortex moves with respect to the other.

A convenient way to look at vortex interactions with a surface is the "image" concept. At a solid surface, there is a no-slip condition. The effect of this is to slow down the vortex flow with respect to the surface; however, this may in fact accelerate the motion of the center of the vortex with respect to the surface. We can model the effect of a solid surface by placing a "mirror image" of the original vortex at a distance from the surface which is correct. For a simple straight wall, the mirror image is just one vortex identical in strength but opposite in sense of rotation to the original vortex, placed at the same distance behind the wall as the original vortex is in front of it. This is shown below:

5. Supersets

6. Subsets

7. Other fields

8. Calculators/Applets

9. 10 Analytical Codes

10. Notes:

11. Byline

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