EXTROVERT

ADVANCED CONCEPT EXPLORATION

ADL P-2011121203

Ryan Donnan, Herman Ryals

Georgia Institute of Technology
School of Aerospace Engineering

Missile Interceptor

December 12, 2011

Publishing Information

We gratefully acknowledges support under the NASA Innovation in Aerospace Instruction Initiative, NASA Grant No. NNX09AF67G, to develop the techniques that allowed such work to be done in core courses, and the resources used to publish this. Tony Springer is the Technical Monitor.

Disclaimer

Missile Interceptor

Ryan Donnan, Herman Ryals

December 12, 2011

At all times, our nation is at risk of attack by an Intercontinental Ballistic Missile. Traditional defense stages include Boost Phase Intercept (lasers), Mid-Course Intercept (Smart Rocks), and Ground-Launched Point Defense System (THAAD). This report explains in detail a new stage, AE3021BF11. This stage consists of a large Transonic Patrol Craft, Uninhabited Combat Air Vehicles, and missiles. The goal of this stage is to intercept the ICBM as quickly as possible, starting from cruise at 40,000 feet.

For our missile defense system, we first designed the Transonic Patrol Craft (TPC). Specifications were mostly obtained from large military aircraft (specifications attached). It carries four Uninhabited Combat Air Vehicles (UCAV), one under each wing and two on the belly of the TPC. The TPC has a cruise speed of Mach 0.6 and cruises at an altitude of 40,000 feet.

The TPC has four turbofan engines that can each produce 55,000 pounds of force at sea level. Using engine design analysis, the following design parameters were calculated to provide maximum fuel efficiency at cruise conditions:

Fuel ratio: 0.01
Bypass ratio: 1
Fan pressure ratio: 1.1
Compressor pressure ratio: 15

Assuming the engine nozzles are perfectly expanded, this would yield a specific thrust of **83.4 (lb_f*s)/lb_m** and a thrust specific fuel consumption of **0.431 lb_m/(lb_f*hr)**. To determine the lift-off distance of the TPC, the following equations were used:

$$K_T = \left(\frac{T}{W} - \mu_r\right) = \mathbf{0.235}$$

$$K_A = -\frac{\rho_\infty}{2\left(\frac{W}{S}\right)}\left[C_{D,0} + \Delta C_{D,0} + \left(k_1 + \frac{G}{\pi e AR}\right)C_L^2 - \mu_r C_L\right] = \mathbf{-7.75 \times 10^{-7}}$$

$$s_g = \frac{1}{2gK_A}\ln\left(1 + \frac{K_A}{K_T}V_{LO}^2\right) + NV_{LO} = \mathbf{6521\ ft}$$

This may seem long, but remember that the TPC is carrying 300,000 pounds of fuel, along with 180,000 pounds of UCAVs and missiles. If the TPC were only carrying half of its fuel, the takeoff distance would be greatly reduced to **4282 feet**. The aspect ratio was calculated to be **7.81** but it needs to be adjusted for compressible flow:

$$AR = \frac{AR_0}{\beta} = \frac{AR_0}{\sqrt{1 - M_\infty^2}} = \mathbf{9.76}$$

To determine the maximum lift-to-drag ratio, we must know the values of the zero-lift drag coefficient and K. Due to additional drag from carrying the UCAVs, the zero-lift drag coefficient is **0.02**. K can be calculated from induced drag. Assuming a spanwise efficiency of 0.9:

$$K = \frac{1}{\pi e AR} = \mathbf{0.0362}$$

$$\left({}^{L}/_{D}\right)_{max} = \frac{1}{\sqrt{4KC_{D,0}}} = \mathbf{18.6}$$

This value can be used to find the maximum endurance of the TPC:

$$E = \frac{1}{c_t}\frac{L}{D}\ln\frac{W_0}{W_1} = \mathbf{20.3\ hr}$$

If the TPC took off with a half-full fuel tank, its endurance would only be **11.3 hours**. In compressible flow, skin friction plays a significant role in overall drag.The first step involves finding the temperature and coefficient of viscosity on the surface of the airfoil. Assuming Mach number and temperature are the same on both sides of the wing:

$$T_{wall} = T^* = T_\infty(1 + 0.1198M_\infty^2) = \mathbf{407°R}$$

$$\mu_\infty = \mu_0\left(\frac{T_\infty}{T_0}\right)^{1.5}\left(\frac{T_0 + 198.72}{T_\infty + 198.72}\right) = \mathbf{2.88 \times 10^{-7}\frac{lb \cdot s}{ft^2}}$$

$$\mu^* = \mu_\infty\left(\frac{T^*}{T_\infty}\right)^{1.5}\left(\frac{T_\infty + 198.72}{T^* + 198.72}\right) = \mathbf{2.98 \times 10^{-7}\frac{lb \cdot s}{ft^2}}$$

Assuming an average chord length of **28 feet**:

$$Re = \frac{\rho V c}{\mu^*} = \mathbf{3.04 \times 10^7}$$

Since this number is well over 1,000,000, we'll use the turbulent skin friction equation:

$$C_f = 0.295\frac{T_\infty}{T^*}\log\left(Re\frac{T_\infty}{T^*}\frac{\mu_\infty}{\mu^*}\right)^{-2.45} = \mathbf{0.00206}$$

Now that we have the skin friction coefficient, we can calculate the drag:

$$C_d = \frac{1}{c}\int_0^c C_{f_u}\,dx + \frac{1}{c}\int_0^c C_{f_l}\,dx = \mathbf{0.00413}$$

$$D_f = \frac{1}{2}\rho_\infty V_\infty^2 S C_d = \mathbf{2283\ lb_f}$$

For the TPC, the drag due to friction is very significant due to the large wing area and relatively high speed. In a worst-case scenario, the TPC will be flying directly away from the ICBM's trajectory. In this case, the TPC will need to rotate 180 degrees before it can release the UCAVs. To determine how long this will take, the maximum angular velocity must be calculated:

$$\omega_{max} = g\sqrt{\frac{\rho_\infty}{W/S}\left[\frac{T/W}{2K} - \left(\frac{C_{D,0}}{K}\right)^{1/2}\right]} = \mathbf{.0201\,rad/s}$$

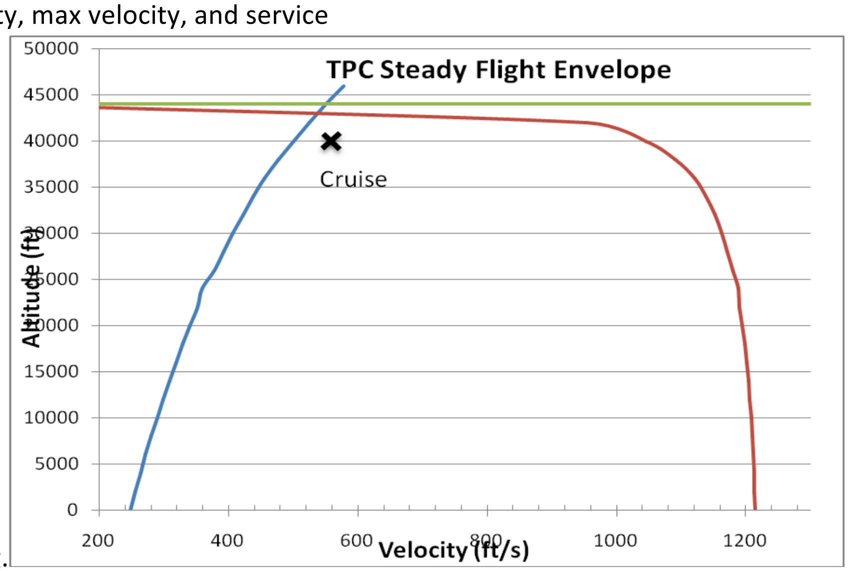
Finally, we divide pi (180 degrees) by our maximum angular velocity to get **156 seconds**. In situations where the TPC can rotate to its desired direction in less than the two minute count down, it will accelerate towards its maximum velocity until the UCAVs have permission to launch. The absolute maximum velocity of the TPC is calculated as follows:

$$V_{max} = \left[\frac{\left(\frac{T_{max}}{W}\right)\left(\frac{W}{S}\right) + \left(\frac{W}{S}\right)\sqrt{\left(\frac{T_{max}}{W}\right)^2 - 4C_{D,0}K}}{\rho_\infty C_{D,0}}\right]^{\frac{1}{2}} = \mathbf{1045\,ft/s}$$

In other words, this is **Mach 1.14**. Since this is almost twice the cruise velocity, it will be assumed that the TPC will not reach Mach 1 before launching the UCAVs. Assuming the critical Mach number of the wings is 0.9, the sweep angle needs to be determined:

$$\Lambda = \cos^{-1}\left(\frac{M_{crit}}{M_\infty}\right) = \mathbf{25.8^\circ}$$

Below is the Steady Flight Envelope of the Transonic Patrol Craft. The velocity is constrained by stall velocity, max velocity, and service ceiling.



The UCAV's propulsion system is modeled after the space shuttle by using 2 outboard-mounted, solid fueled rockets along with its 4 internal liquid fueled rockets. Each solid rocket motor is a two stage rocket and produces 25,000 pounds of thrust. The liquid fueled rocket motors produce up to 5,000 pounds of thrust each for a combined total of 20,000 pounds of thrust. Four hypersonic missiles are mounted on the wings and are to be launched in the case of an incoming ICBM. The 4 internal rockets will initially power the aircraft up to 50,000 feet, or about 90 seconds. Next, the 2 outboard motors will kick in and accelerate the UCAV up to Mach 4 and 150,000 feet, taking about 90 seconds as well. Since the UCAV's engines aren't air breathing, the aircraft technically does not have a service ceiling. At the same time, it does require a significant amount of air for stability and maneuverability, so it is designed to fly up to 150,000 feet even if it can fly higher.

When the aircraft is launched from TPC it will be oriented at 10 degrees relative to the horizon. Onboard stabilization and attitude control systems will enable the aircraft to maintain this orientation. After engine ignition, and during both stages, the UCAV will be at a constant angle of attack of 3 degrees, constantly increasing the slope of its trajectory to increase the rate of climb.

To determine the drag coefficient, the thickness distribution of the UCAV is approximated by the following equation:

$$y = \frac{2tx}{c^2}(c - x)$$

The thickness to chord ratio of our UCAV is 2%. The following equation is then used to calculate the drag coefficient:

$$C_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{2}{\sqrt{M_\infty^2 - 1}} \int_0^1 \left[\left(\frac{dy_u}{dx}\right)^2 + \left(\frac{dy_l}{dx}\right)^2 \right] d\left(\frac{x}{c}\right)$$

Assuming average velocity during the first stage is Mach 1.3, the drag coefficient will be roughly **0.023** for that stage. If the average velocity is Mach 3 for the second rocket stage, the drag coefficient will be **0.0074**. As the UCAV approaches 150,000 ft and Mach 4, drag on the aircraft will only be **48 pounds** due to the extremely low density. Skin friction drag is also rather insignificant due to the low density, adding only **38.8pounds**of drag at Mach 4.

By using the approximation from thin airfoil theory, the lift coefficient at this angle of attack can be calculated from the following:

$$C_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

From this equation our coefficient of lift is **0.26** at our assumed average speed of Mach 1.3. As we increase in Mach number and altitude the lift coefficient reduces to **0.073** at Mach 3. Finally, when the UCAV reaches its maximum velocity at Mach 4 the coefficient is only **0.054**. This results in a lift of **488 pounds**. Our **average L/D** for this flight trajectory is just over 10.3. This may seem to be a little high at first, but this is the nearly ideal case with the assumptions we have made so a high estimate is the expected result.

The pitching moment at supersonic speed can be calculated from the following equation:

$$C_m = -\frac{4\alpha}{\sqrt{M_\infty^2 - 1}}\left(\frac{1}{2} - \frac{x_0}{c}\right) + \frac{4}{\sqrt{M_\infty^2 - 1}}\int_0^1 \left(\frac{dy_c}{dx}\right)\frac{x - x_0}{c} d\left(\frac{x}{c}\right)$$

However, since we are assuming our airfoil is symmetric and at supersonic speed, the moment about the half chord point is zero. Implementing the same assumptions used for calculating the drag coefficient yields a pitching moment coefficient of **0.0049** at Mach 1.3. As the UCAV approaches Mach 4 and the peak of its flight, the coefficient drops all the way to **0.0012**.

In order to calculate the coefficient of lift for the duration of the UCAV's flight that is subsonic we needed to use slender wing theory to transform our wing for transonic conditions. The equation for lift coefficient in incompressible flow is transformed below.

$$C_l = \frac{\pi}{2}(AR_0)\alpha$$

where $$AR_0 = AR\sqrt{1 - M_\infty^2}$$

Using this transformation at Mach 6, which is the initial speed when the UCAV is launched, we obtain a lift coefficient of **0.14**. However, as the UCAV approaches Mach 1, the value of C_l approaches 0 using this equation. At Mach 0.9 the coefficient is only **0.075**. The slender wing transformation becomes an invalid estimate near Mach 1.

As the UCAV nears Mach 1 the drag increases rapidly due to wave drag. At lower speeds the induced drag is the driving drag force, but it varies inversely with Mach number. For our estimates drag is composed of only wave drag and induced drag. These values can be found from:

$$C_{Di} = \frac{C_l^2}{\pi AR}$$

$$C_{Dw} = \frac{4\alpha^2}{\sqrt{1 - M_\infty^2}}$$

The total coefficient of drag is the sum of these two values. At Mach 0.6 with a 3 degree angle of attack the drag coefficient is **0.0137**. This gives us a value for L/D slightly above 10, just like at supersonic conditions. However, at Mach 0.9 the drag coefficient is **0.025**. This results in an L/D of 5.6. This shows why it is recommended to spend as little time around Mach 1 as possible as the drag begins to increase very rapidly around this flight condition.

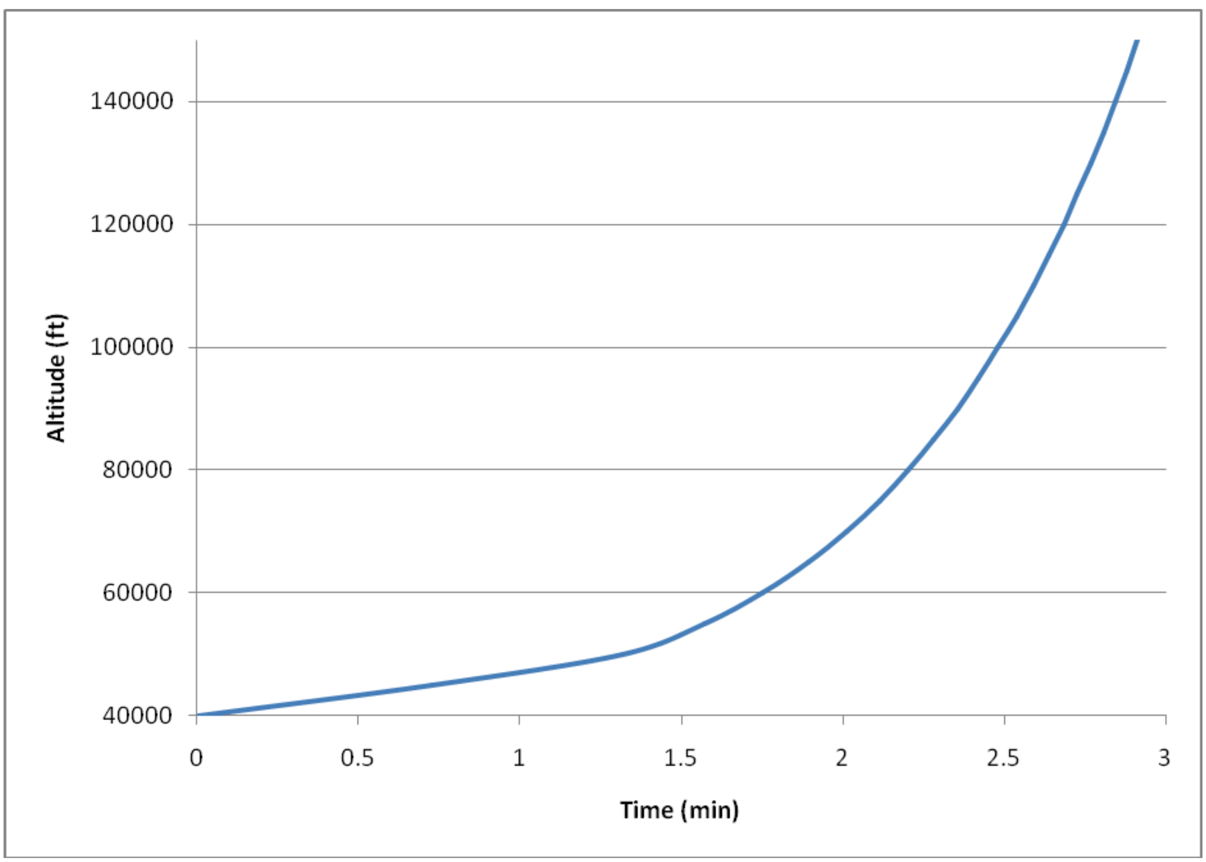
To determine the relationship between time and altitude, a major assumption was that the UCAV would increase its angle of flight by 1 degree every 5000 feet. The next step was to approximate the acceleration of the aircraft based on drag, thrust and gravity:

$$Acceleration = \frac{F}{m} = \frac{T - D}{m} - g\sin(\theta)$$

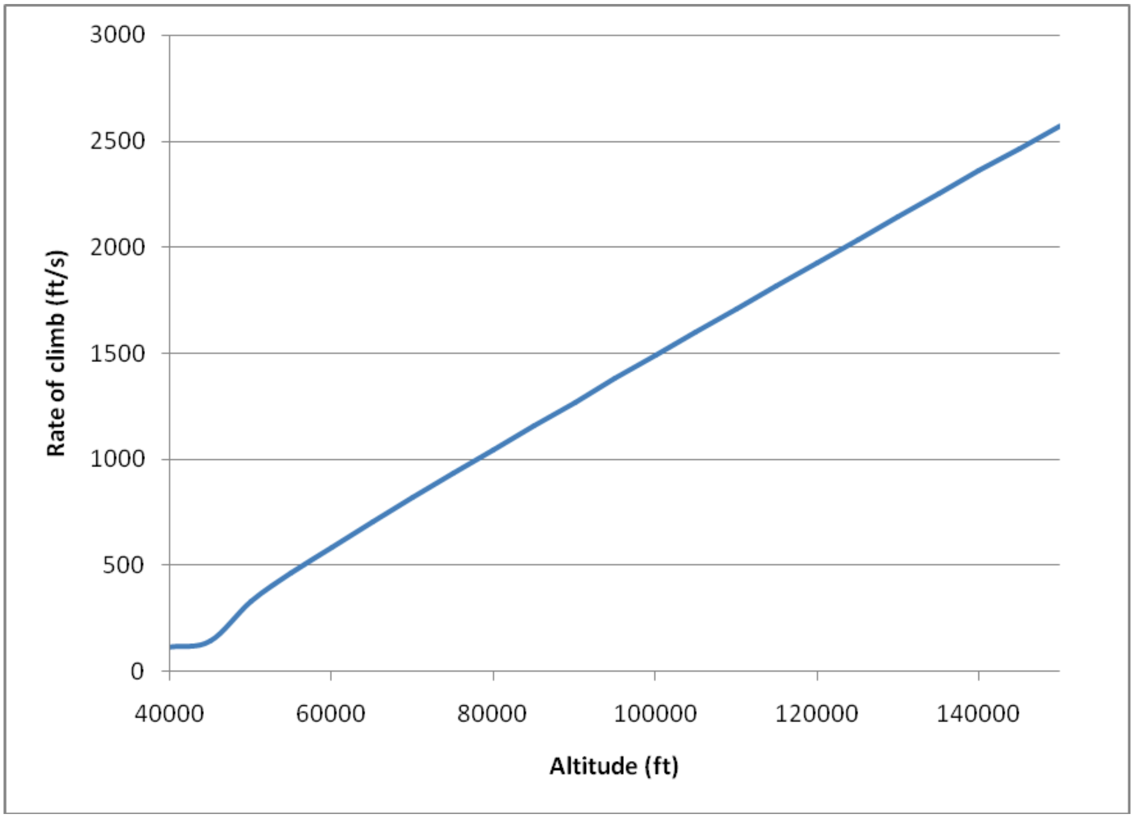
In order to know the magnitude of drag at the next height increment, velocity is calculated from acceleration and distance traveled:

$$v_f^2 = v_i^2 + 2ad = v_i^2 + 2a \times \frac{5000}{\sin(\theta)}$$

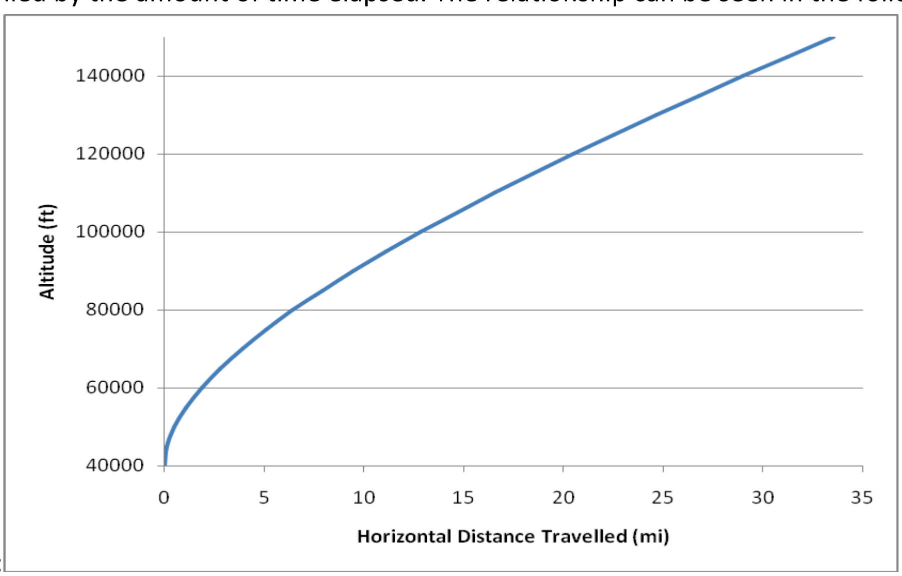
Finally, time is calculated by dividing distance travelled by approximated velocity. Using this strategy, it was determined that stage one takes 1.31 minutes and stage two takes 1.63 minutes, yielding a total climb time of about **2.94 minutes**.



The UCAV climbs much faster at higher altitudes due to higher flight angle (shorter distance required to climb), lower density, and higher thrust in the second stage. The instantaneous rate of climb is calculated by multiplying the tangential velocity of the UCAV by the sine of its flight angle. The following plot shows the relationship between altitude and rate of climb.



The sudden change in slope of the curve signifies the transition from stage 1 to stage 2. To find out how far away the UCAV would end up from its original launch point, the horizontal portion of velocity was multiplied by the amount of time elapsed. The relationship can be seen in the following graph:



By the time the UCAV reaches 150,000 feet, it will have traveled **167 miles**. At that point,the solid rocket motors will detach from the UCAV two at a time and boost the missiles away from the UCAV. The UCAV then glides back to sea level (similarly to space shuttle reentry), using small parachutes for deceleration and stability. Based on the UCAV's lift to drag ratio, the minimum glide angle can be approximated to be around 7 degrees. This allows the UCAV to glide for over 200 miles after it reaches the top of its trajectory, which is more than enough to reach its final destination.

The missile is a 6 ft hypersonic scramjet mounted on top of a 3 ft solid rocket motor. The rocket fires first and is used to boost the missile far ahead of the UCAV that launched it. Once the rocket has used up all of its fuel, a charge will fire and detach the rocket from the scramjet section. The scramjet engine then takes over, allowing the craft to reach Mach 10. The missile is fitted with a sophisticated GPS and tracking system that allows an incoming threat to be immediately located and targeted. Since the ICBM will be in freefall at this point, not much maneuverability will be required. The scramjet will accelerate towards the incoming missile and intersect it head on around 200,000 feet.

The missile kills, or disables, an incoming enemy missile kinetically. In the nose of the missile is an explosive, shotgun-like weapon that fires when in range of the target. It sprays buckshot-like projectiles in the path of the incoming target and tries to intersect the missile itself. The enemy ICBM will make contact with one, or both, and be destroyed. Our missile will explode on impact. But, if the missile misses the target it will still detonate itself midair, shortly thereafter.

The lift and drag of the missile are calculated from Newtonian Aerodynamic principles and Modified Newtonian Aerodynamics. This means that the momentum of the flow is split into vertical and horizontal components and transferred to the aircraft as lift and drag. First, the pressure coefficient must be calculated using Newtonian Aerodynamics:

$$c_p = \frac{\frac{p_{02}}{p_\infty} - 1}{\frac{\gamma}{2} M_\infty^{\ 2}}$$

where

$$\frac{p_{02}}{p_\infty} = \left[\frac{(\gamma+1)^2 M_\infty^{\ 2}}{4\gamma M_\infty^{\ 2} - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1-\gamma+2\gamma M_\infty^{\ 2}}{\gamma+1}\right]$$

From these equations the pressure coefficient is found to be **0.19**. This value is then split into lift and drag components using trigonometric functions to yield **0.18** and **0.045** respectively. This results in an L/D of 4. As Mach number increases, the pressure coefficient decreases. Thus, the lift and drag coefficients decrease as well. However, since they are both trig components of the pressure coefficient, they scale symmetrically and the L/D remains constant.

Using the hypersonic shock and expansion relations we can find the shock angle β needed to produce these values. Plugging the pressure coefficient into the following equation and solving for β will yield the desired angle.

$$C_p = \frac{4Sin^2\beta}{\gamma + 1}$$

From this relation we obtain **β** to be **19.7 degrees**. This is the angle a shock makes, with respect to the direction of the flow, after coming in contact with the bottom of the scramjet. To determine the angle of attack we need to fly at to produce the calculated values, we substitute β into this equation:

$$\theta \cong \left(\frac{2\beta}{\gamma + 1}\right)$$

Using this approximation, Θ is found to be 16.4 degrees. So, for our hypersonic missile the angle of attack we need to fly at to produce an L/D of 4 is 16.4 degrees.

However, these calculations only account for induced drag. Wave drag and viscous drag are also components of the total drag. In order to account for this, we used R.T. Jones's equation for ellipses to find the volume-dependent wave drag.

$$C_{D_w} = \frac{\pi l^2}{16S} C_L^2 \left[\sqrt{1 + (M^2 - 1)\left(\frac{4S}{\pi l^2}\right)^2} - 1 \right]$$

Using Mach 7 as the average Mach number during our hypersonic flight wave drag was found to be **0.025**. Adding this value to the induced drag results in an overall drag of **0.070**. Our L/D is now only 2.57. As it can be seen from our calculations, at hypersonic speeds wave drag is large factor in the overall drag and needs to be taken into account.

This entire process—beginning with the turning of the TPC and ending with the hypersonic missile reaching the ICBM at 250,000 feet—will take between 4 and 5 minutes, depending on the location of the TPC. The current THAAD ground-launched system takes over 5 minutes, not including the 2 minute countdown. Our system is clearly a better defense mechanism, not only because it takes less time, but because a majority of the components are recoverable. Our system is also protected from false alarms. Even though nothing like this system exists, all of our components are based on real vehicles and aerodynamic principles.

	TPC	UCAV	Hypersonic Missile
Length	240 ft	25 ft	6 ft
Wingspan	220 ft	23.5 ft	6 ft
Height	65 ft	5 ft	0.5 ft
Wing Area	6200 ft^2	250 ft^2	9 ft^2
Payload	220,000 lb	4000 lb	None
Empty Weight	360,000 lb	19,000 lb	1000 lb
Max Takeoff Weight	800,000 lb	45,000 lb	1000 lb
Power Plant	4 x 55,000 lb turbofan engines	4 x 5000 lb liquid fuel rockets and 2 x 25,000 lb solid fuel rockets	Scramjet
Service Ceiling	46,000 ft	160,000	250,000
Cruise Speed	Mach 0.6	Up to Mach 4	Up to Mach 10
Range	7500 mi	500 mi	400 mi