On the Flowfield and Forces Generated by a Flapping Rectangular Wing at Low Reynolds Number

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Abstract
Measurements are reported of the velocity field, vorticity field, and forces generated by a rectangular flat plate wing with 3% circular arc camber and aspect ratio 2.42. The wing is flapped at a reduced frequency of 0.5 with flapping amplitude of +35.2°/-13.4° and chord Reynolds number 20,000. The periodic velocity field is measured by laser velocimetry over a grid of approximately 0.05c spacing. Experimental data are compared to potential flow models to highlight issues in the low-Re, highly unsteady flight regime.

Nomenclature
$C_L$ Lift coefficient, $= L/(1/2 \rho U_{\infty}^2 S)$
c wing chord length
d distance from wing pivot to center of mass
$F_{\text{meas}}$ total measured force
L Lift
m mass of wing
t time
S wing area
T flapping period
$u, v$ x, y components of velocity, respectively
U velocity magnitude
$U_{\infty}$ freestream velocity
$\alpha$ wing angle of attack
$\gamma$ wing angular acceleration, $= d\Omega/dt$
$\theta$ flapping angle
$\Omega$ wing angular speed, $= d\theta/dt$
$\omega_z$ z component of vorticity

superscript dot denotes differentiation with respect to time
overbar denotes average

Introduction
This paper describes an experimental study of the flowfield and forces produced by a flapping flat plate where the amplitude of the flapping is large, the reduced frequency is moderate, and the chord Reynolds number is low. The objective of the work is to generate a basic experimental test
case that can be used to connect and validate the results on unsteady aerodynamics over a wide range of Reynolds number and unsteadiness.

The aerodynamics of small-amplitude wing motions in the reduced frequency range 0<k<0.1 is well understood for applications where the chord Reynolds number is large. Many applications for autonomous flight vehicles in the atmospheres of Earth and other planets occur in the Reynolds number range below 100,000; some are as low as 10,000. Several of these applications require very small vehicles, low flight speeds, and high levels of unsteady motion due to either gusts or quick maneuvers. Vehicle designs in this range may benefit from the use of wings flapping with high amplitudes and frequencies.

Past work in this area has been driven by interest in fixed-wing and rotary wing uninhabited aerial vehicles, large-scale ornithopters, and in the flight mechanisms of actual birds and insects. The aerodynamics of all but the smallest of birds, and of insects, appears to be predictable using aerodynamic theory developed for larger aircraft. Thus, steady and quasi-steady analyses using panel codes appear to suffice for fixed-wing UAVs and the larger birds where the reduced frequency of wing motion is small and the flight Reynolds number is on the order of 50,000 or more. Attention has been devoted to airfoil aerodynamics at low Reynolds number and low to moderate angles of attack.

**Previous Work**

There is as yet only a small amount of research that has been directed towards the problem of low-Reynolds number, unsteady flows. Most of the work related to the low-Reynolds number flight regime has been done with regard to design and optimization at low angles of attack and steady flight conditions. The majority of the work in the field of unsteady aerodynamics has been directed at higher-Reynolds number flows. There is a vast amount of research directed toward the measurement and prediction of dynamic stall and of airfoil/wing behavior for small excursions around some nominal angle of attack. This work has been directed at developing the ability to predict such phenomena as flutter, aircraft gust response, and pitch rate effects (with particular emphasis on helicopter aerodynamics). Figure 1 gives a graphical description of the existing general aerodynamic knowledge database in terms of reduced frequency and Reynolds number.

**Numerical Studies**

*General Two-Dimensional Flows*

The well-known lift-deficiency theory due to Theodorsen is probably the most-widely applied theory in the field of unsteady aerodynamics. However, inherent within this theory is the assumption of infinitely thin regions of vorticity that are easily separated into bound circulation on the airfoil and that contained within a planar wake. While appropriate to attached, high-Reynolds number flows where the effects of vorticity are generally contained within the thin airfoil boundary layer, the increased (and widespread) effects of viscosity at low Reynolds number brings to question the applicability of such a theory.

Using a numerical approach based on singularity distributions provides a means by which airfoil thickness can be accounted for and the planar wake assumption can be removed. However, the assumption of a singular vorticity distribution still confines the effects of vorticity
to narrow regions around the airfoil and the wake. Full Navier-Stokes calculations have also been performed for low-Reynolds number cases, removing all assumptions about vorticity confinement. However, the computational time required for these numerical investigations is prohibitive due to the low-Re and combined laminar/turbulent nature of the free shear layers generated.

**General Three-Dimensional Flows**

There has been significant work in expanding the results of two-dimensional unsteady aerodynamics into the third dimension, even at the level of Navier-Stokes computations. Much of this work has been an extension of methods applied to helicopter aerodynamics and, as such, has been limited to high-Reynolds number flows. Most of these models (all numerical, none analytical) have fallen into three categories: blade-element-type computations where the three-dimensional nature of the body is represented by a collection of two-dimensional entities; lifting-line computations that replace wings and rotors with filaments of vorticity bound along the span; and singularity distributions over a paneled representation of the body (panel codes).

These unsteady numerical models have been developed both in the time domain and the frequency domain and even for separated flows, but all share the same high-Reynolds number restrictions. There have been examples of the application of these methods to unsteady, low-Reynolds number flight, but the results should be viewed in the light of the mismatch between the assumptions behind the computations and the flight vehicle/conditions modeled.

**Experimental Studies**

The difficulties associated with low-Reynolds number aerodynamic testing are well documented, as are the difficulties associated with unsteady measurements. Briefly, most experimenters have encountered serious problems with repeatability and facility-dependence of lift and drag measurements at Reynolds numbers below 300,000. This is usually attributed to the critical dependence of transition lines, separation lines and separated flow regions on various parameters which are difficult or impossible to control adequately. The vast body of work done by the NACA in measuring force and moment coefficients for various airfoils was limited to Reynolds number greater than 300,000. Much work since then has been done to extend this body of experimental measurements to the low-Reynolds number regime. However, these measurements have all been directed at steady flows.

The mechanisms of bird and insect flight have also been studied with flow visualization techniques. A limited number of force measurements have been made for low-Reynolds number flapping wings. Higher-Reynolds number force measurements on flapping wings have been done with primary emphasis on thrust development.

**Scope of Present Work**

Referring to Figure 1 and the previous work, some comments about the terms “high” and “low” in the context of reduced frequency and Reynolds number, respectively, would serve to set the present work in context. Textbooks on unsteady aerodynamics generally define the ranges of unsteadiness in terms of the Theodorsen function and the regimes where various assumptions hold. Thus, $0 < k < 0.03$ is nominally considered “quasi-steady” (wake effects not very significant), $0.03 < k < 0.1$ can be considered “quasi-unsteady” (wake effects significant but apparent-mass
acceleration effects negligible) and beyond that is considered “fully unsteady” (all unsteady effects important). For k>1, acceleration effects begin to dominate. Figure 1 shows that little is really known about flowfields beyond a reduced frequency of 0.5. Insect flight, and the wing-flapping of hummingbirds are beyond this range. It is apparent that airfoil aerodynamics below a Reynolds number of 300,000 poses problems, and below 50,000, little is known at any significant level of unsteadiness. These help define the parameter range of the present experiments. The Reynolds number is kept well below 50,000. The reduced frequency is kept in the range from 0.1 to 0.6; the present data are at 0.5. This allows comparisons with theory to begin with potential-flow methods with some reasonable expectation of being able to identify similarities and issues. Even here, the flapping rate is sufficiently high to induce instantaneous angles of attack which are well beyond the attached-flow regime.

**Experimental Setup**

Velocity measurements were carried out using a laser velocimetry system (LV) in the John J. Harper Wind Tunnel at Georgia Tech. The closed-return tunnel has a 7’ x 9’ atmospheric-
pressure test section and was run at a freestream speed of 10 ft/s, giving a chord Reynolds number of approximately 20,000.

The LV system used a 6-watt Argon ion laser with the light transmitted and received through a fiber-optic probe. The probe was installed on a two-axis traverse with a position accuracy of better than 0.001” and was controlled via a serial interface to a Pentium PC. The velocity was measured using a counter/processor interfaced to the PC. The LV counter processor was configured to use 32 cycles/burst with 1% comparison to define a data point. A once-per-cycle timing pulse from the wing mechanism provided the phase reference to sort the velocity data. The data were sorted into 500 bins per cycle and ensemble-averaged over approximately 120 data points per bin. The root-mean-square velocity fluctuation in each bin was also computed. The velocity measurements were performed one component at a time.

The wing model (shown in Figure 2) had a rectangular planform spanning 10” with a chord of 4.125”. The wing was constructed of 1/16” sheet balsa wood with 1/32” sheet aluminum reinforcement near the root. The wing was covered with Monokote™. The wing had a circular arc camber of 3 percent, concave to the x-axis. The reference axes were defined as shown in Figure 2 with the wing in the vertical position. Positive flapping angles are given by the right-hand rule in the x-y plane, measured from the y-axis. The freestream is in the +z direction.

<table>
<thead>
<tr>
<th>Planform: Rectangular plate</th>
<th>Freestream Speed: 10ft/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chord: 4.125”</td>
<td>Flapping Frequency: 4.5 Hz</td>
</tr>
<tr>
<td>Span: 10.00”</td>
<td>Amplitude: +35.2°/-13.4°</td>
</tr>
<tr>
<td>Camber: 3% circular arc</td>
<td>Reduced Frequency: 0.5</td>
</tr>
</tbody>
</table>

**Wing Motion**

A MicroMo DC servomotor with 76:1 gearbox provided the flapping mechanism. The reduced frequency is given by

\[ k = \frac{\pi f c}{U_{\infty}} \]

Using this definition gives \( k = 0.5 \) for the case where the flapping was at 4.5Hz. The flapping motion was approximately sinusoidal (as shown in Figure 3) with a maximum at +35.2° and a minimum at -13.4°. Figure 3 also shows the wing angle of attack, angular velocity, and angular acceleration. The master once-per-cycle trigger fired at \( T_0 = 0 \)s corresponding to a wing position that is just short of its maximum leftward (-x) position. This position also corresponds to the time just before the wing begins the down stroke.

Wing position data were obtained using a US Digital optical encoder and a Technology 80 TE5312 encoder counter. The encoder uses a 1024 count/rev disk in quadrature mode, giving 4096 counts/rev or a precision of +/-0.088°. Flexibility in the wing is not accounted for; strobe light visualization showed no evidence of significant motion-induced bending.

Wing angle of attack, angular velocity, and angular acceleration were calculated based on the wing position measurements. The instantaneous angular velocity was computed based on the
encoder data and used to compute the motion-induced velocity at 80% span. Defining angle of attack as

$$\alpha = \tan^{-1}\left(\frac{v_i}{U_{\infty}}\right)$$

where \(v_i\) is the motion-induced velocity gives the angle of attack variation shown in Figure 3. As shown in the plot, the angle of attack at 80% span ranges +/- 38°.

**Velocity Data Planes**

The velocity and flow visualization data were taken in five planes. These planes were in the plane of flapping (x-y plane) at the leading edge, the wing mid-chord, the trailing edge, one chord length, and two chord lengths downstream of the trailing edge.

The measurement points in the first three planes were spaced at 0.048c in the x direction and 0.061c in the y direction (see Figure 4). These planes spanned 1.21c in the y direction and 2.91c in the x direction, giving a total of 1281 measurement points per plane. The (0,0) point was placed at the tip of the wing in the 0° position and the measurement plane extended 0.67c above, 0.55c below, and 1.45c to either side. The wake measurement planes used the same zero reference point but extended 1.45c above, 0.97c below, 2.42c in the –x direction, and 1.45c in the +x direction, giving 693 measurement points per plane. Grid spacing was 0.061c in both the x and y directions in the wake measurement planes.

**Velocity Field Data Analysis**

The measured velocity data were sorted into 500 bins per cycle and were used to numerically compute the in-plane component of vorticity, given by

$$\omega_z = \frac{1}{2} \left( \frac{dv}{dx} - \frac{du}{dy} \right)$$

Contours of the in-plane vorticity component were animated to view the dynamic flowfield. The mean flowfield values were computed by averaging over all bins for a particular measurement point. Vortex core positions were plotted by locating the center of rotation from vector plots of the LV data.

The root-mean-square (rms) velocities were also computed for each bin of each LV grid location. The rms velocity is given by

$$U_{rms} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (U_i - \overline{U})^2}$$

where \(n\) is the total number of data points taken for a given bin, \(U_i\) are the velocity measurements, and \(\overline{U}\) is the ensemble average for all \(n\) data points in a given bin. Note that \(U_{rms}\) is the rms velocity for a particular bin at a particular grid location. As a result, for a given grid location, the value of \(U_{rms}\) will (in general) vary in time.

**Force Measurements**

Force measurements were obtained by mounting the entire flapping wing assembly onto a two-component force/moment balance. The data were recorded and stored using a National Instruments analog-to-digital conversion system. The once-per-cycle timing reference was used to trigger the acquisition in order to provide a phase reference for the force measurements. In this
experiment, the flapping rates and freestream speeds were varied over a wide range. The wing motion histories changed from their intended sinusoidal nature at the higher speeds, as the load on the motor varied through the cycle. Actual motion histories detailed in Ref. 31 were used in the panel code computations described later in the paper.

At the high flapping rates, the inertial loading on the force balance was a significant portion of the total measured force. As a result, the following dynamic model was used to correct the measured force data:

\[
L = F_{\text{meas}} - m d [\dot{\theta} \sin(\theta) + \dot{\theta}^2 \cos(\theta)]
\]

where \(F_{\text{meas}}\) is the total measured force, \(L\) is the aerodynamic force (lift), \(m\) is the mass of the wing, \(d\) is the distance from the wing pivot to its center of mass, and \(\theta\) is the measured flapping angle. Inertial loading due to unsteady flex in the force balance was not considered.

**Results and Discussion**

The plots in Figures 5 – 9 give the normalized velocity and vorticity field measurements for the leading edge, mid-chord, trailing edge, 1c downstream, and 2c downstream planes, respectively. Figure 10 gives the normalized rms velocity contours in the 2c downstream plane. The velocity vectors and rms velocities were normalized to \(U_{\infty}\) and the vorticity values were normalized to \(U_{\infty}/c\). This collection of plots is necessarily limited due to the print medium; a complete collection is given in Ref. 31. Figure 15 gives a matrix of plots showing the measured and computed unsteady lift coefficients for various combinations of freestream speeds and flapping frequencies.

**Velocity Data**

There are four characteristics of the velocity field that are of note: first, the nature of the vorticity concentrated within the tip vortex is unlike that of a high-Re, steady fixed-wing vortex. Rather than concentrated circular tip vortices, we see separated shear / vorticity layers forming well away from the nominal center of rotation, resembling the situation over a delta wing at high angles of attack. Second, the tip vortex becomes very far removed from the wing tip at the trailing edge plane, moving as far as 0.75c away from the tip of the wing. The outward motion of the vortex is attributed to the centrifugal acceleration of the flow as the wing rotates through its flapping motion. Third, the vortex “core” is stretched in the direction of flapping, particularly at the mid-chord and trailing edge planes. Finally, the rms velocities in the wake are much higher than what would be expected from a low-Re (presumably laminar) flow. This is attributable to the development of discrete vortical structures in the separated shear layers (see Fig. 10).

In the mid-chord and trailing edge planes (Figures 6-7), the areas of concentrated vorticity more closely resemble a shed vortex sheet rather than a single, concentrated vortex. The vorticity tends to be positioned mostly to one side of the core, a fact that is manifested in a highly asymmetric velocity distribution across the vortex core (with much higher velocities on the “vorticity” side of the core).

This asymmetry exists far away from the wing, as well. As shown in Figure 11 (which shows circumferential velocity across the trailing tip vortex 5c downstream), the velocities on one side of the vortex core are nearly five times as large as those on the other. Furthermore, the core region, i.e., the region of increasing circumferential velocity, extends from approximately –0.30c...
to 0.25c, giving a core diameter of more than 0.50c, substantially larger than the 0.03c-0.08c typical of higher-Re tip vortices.

The velocity field data in Figure 7 show that the tip vortex becomes very far removed from the wing tip as the wing begins on the down stroke. The center of rotation stays in very nearly the same location for t/T = 0.2 to 0.4 while the wing is pulling away on the down stroke. Figure 12 shows the normalized distance from the tip of the wing to the center of the tip vortex at the mid-chord and trailing edge planes. As shown in the plot for the trailing edge data, the wing tip moves nearly 0.75c away from the tip vortex before it begins to catch up. The same phenomenon is observed on the up stroke, as well, and in the mid-chord plane, but the effect is not nearly as pronounced as it is at the trailing edge on the down stroke.

As the tip vortex begins to catch up to the wing tip, the core appears to be “stretched” in the direction of flapping. As shown in Figure 13, the region that would normally be considered the “inner core region” of the tip vortex is stretched in the direction of flapping from approximately –0.70c to –0.20c. This core-stretching phenomenon is not apparent in any of the other data planes and does not appear downstream. It has to be attributed to distortions in the flowfield due to the unsteady accelerations of the flow, but beyond that, it has to be left to flowfield computations to identify the precise causes and effects.

The normalized rms velocities in the 2c downstream plane are shown in Figure 10. The plots show large regions where the magnitude of the velocity fluctuations approach 40% of the freestream speed. A comparison between Figures 10 and 9 shows that the areas of high fluctuation almost exactly follow the areas of concentrated vorticity. Because these values are extraordinarily high, they have been validated against hot-film measurements at several locations. Such high values can only result from the rollup of discrete vortical structures in the shear layer. Such structures would have a range of sizes, and involve kinetic energy over a broad range of frequencies. There is no reason to believe that these vortical structures will bear an integer relationship to the flapping frequency, hence these velocity fluctuations will appear as perturbations in the phase-resolved velocity measurements. The Reynolds number is too low, of course, to attribute these to random turbulence.

**Force Data**

As noted above, the inertial loading due to the motion of the wing was a substantial portion of the total measured force. Figure 14 shows a breakdown of the force components for a typical flapping cycle at 4.5Hz and 10 ft/s freestream speed. The plot shows that the inertial loading is highest at the start of the flapping cycle, peaking at more than 50% of the aerodynamic force shortly after the down stroke has begun. The asymmetry in the inertial loading is a consequence of an asymmetric flapping setup in the wind tunnel and the slightly non-harmonic flapping motion. At the higher tunnel speeds used in these experiments, the motion histories became substantially distorted (see Ref. 31 for details).

The force measurement data were compared against computations from an unsteady panel code. The flapping wing model was simulated in an unsteady vortex lattice code with a free wake generated at the trailing edge panels. The code used 40 panels to represent the wing geometry and 300 time steps over two flapping cycles for the various combinations of freestream speed.
and flapping frequency. The influence of the free wake on the wing and on itself was calculated for the 75 most recent time steps. Wake panels older than 75 time steps were neglected. The wing motions used in the panel code were based on curve fits of the measured wing motion. Thus, higher-frequency unsteadiness within a cycle would be underpredicted by the code. Results from the second cycle of the wing motion are used in Figure 15, to reduce the effects of the starting vortex at the impulsive start of the computations.

Figure 15 shows cases at two extremes of the reduced-frequency / Reynolds number ranges studied. Figure 15 (a) shows results at a flapping frequency of 4.5 Hz, and freestream velocity of 10 fps. This combines the highest reduced-frequency and lowest Reynolds number studied. Here the wing motion is largely simple harmonic, but the experimental result shows large fluctuations. The experimental results have not been averaged over many cycles, so that fluctuations which are not synchronized with the flapping phase are also captured. The panel code greatly underpredicts the force variation. The matrix of plots in Figure 15 shows two competing trends: for a given freestream speed, as the flapping frequency increases, the unsteady panel code computations under-predict the force generated by the wing. However, in Figure 15(b), the agreement between calculation and experiment is seen to be quite good. Here the freestream velocity is 60 fps, the highest studied, and the flapping frequency is 1 Hz, the lowest reduced-frequency case studied. The wing motion here is not simple harmonic: the cambered wing experiences substantially more force during motion to one side than to the other, and the motor is unable to overcome this asymmetry completely. The panel code overpredicts the force slightly in this case.

There is a substantial amount of higher harmonic content in the force measurements than predicted by the panel code, particularly on the down stroke. This type of fluctuating force hints at some type of shedding off the wing and is also evidenced in the LV data in the 2c downstream plane. Here, discrete patches of negative vorticity are seen circulating around a vortex core from $t/T = 0.19$ to $0.32$ (though it should be noted that this vorticity is associated with the upstroke). This aspect is not understood at present. From the above, it appears that the panel code is an adequate representation of the latter test case, but performs poorly in the former. At the limit of high reduced frequency and low Reynolds number, the flowfield is dominated by large instantaneous angles of attack, separated flows, smeared distributed regions of vorticity, and shear layers with discrete vortical structures, none of which can be expected to be captured in the panel code.

**Conclusions**

This paper has presented quantitative velocity field data and panel code computations for a rectangular wing undergoing large amplitude flapping at moderate reduced frequency and low chord Reynolds number. The observed characteristics of the velocity field include the following:

1. Vorticity is distributed over the tip vortex, unlike what is seen in vortices at higher Reynolds number.
2. The tip vortex cores become stretched in the direction of wing flapping during the downstroke.
3. The velocity distribution across the vortex core is highly asymmetric.
4. Predictions from an unsteady panel code approximate measurements at the test case of highest Reynolds number and lowest reduced frequency.
5. At higher reduced-frequency and low Reynolds number, the panel code underpredicts force by an order of magnitude.
6. The lift exhibits substantial higher-frequency spectral content; these are attributed to the generation of discrete vortical structures in shear layers and near the concentrated vortices due to unsteady motion.

References
Figure Captions

Figure 1: Existing general aerodynamic knowledge database in terms of reduced frequency and Reynolds number.

Figure 2: Wing Configuration. Freestream is along the z-axis.

Figure 3: Wing flapping angle ($\theta$), angle of attack ($\alpha$), angular velocity ($\Omega$), and angular acceleration ($\gamma$).

Figure 4: Grid points for LV measurements: (a) leading edge, mid-chord, trailing edge; (b) wake. Thick line shows wing in zero degree position.

Figure 5: Normalized velocity/vorticity field measurements in the leading edge plane. Wing indicated by thick dark line. Unit vector shown in upper left of each plot; time stamp shown in upper right.

Figure 6: Normalized velocity/vorticity field measurements in the mid-chord plane. Wing indicated by thick dark line. Unit vector shown in upper left of each plot; time stamp shown in upper right.

Figure 7: Normalized velocity/vorticity field measurements in the trailing edge plane. Wing indicated by thick dark line. Unit vector shown in upper left of each plot; time stamp shown in upper right.

Figure 8: Normalized velocity/vorticity field measurements in the 1c downstream plane. Wing indicated by thick dark line. Unit vector shown in upper left of each plot; time stamp shown in upper right.

Figure 9: Normalized velocity/vorticity field measurements in the 2c downstream plane. Wing indicated by thick dark line. Unit vector shown in upper left of each plot; time stamp shown in upper right.

Figure 10: Normalized velocity/rms velocity field measurements at 2c downstream. Wing indicated by thick dark line. Unit vector shown in upper left of each plot; time stamp shown in upper right.

Figure 11: Normalized circumferential velocity across the vortex core in the 5c downstream plane. Data are taken from the t/T = 0 bin.

Figure 12: Tip vortex core distance from wing tip (normalized)

Figure 13: Stretched vortex core indicated by double-arrowhead line in the trailing edge plane. Wing motion is left to right at t/T=0.376; instantaneous angular velocity is 578°/s. Reference unit vector shown in upper left.

Figure 14: Unsteady lift, inertial loading, and aerodynamic force in lb

Figure 15: Measured and computed force data, and a curve fit to the motion history, for (a) 10fps, 4.5Hz, and (b) 60fps, 1.0Hz.
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