Aerodynamic Missile Defense System

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Publishing Information

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AE-3021
HIGH SPEED
AERODYNAMICS
FINAL PROJECT REPORT

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Introduction

One of the most important determinant factors involved in national security can be considered to be strategy. It has played a huge role for many centuries. Furthermore, Strategic deterrence can be defined as a military strategy under which one power uses the threat of reprisal effectively to preclude an attack from an adversary power. This simply means that if there was anyone considering the chances of launching a devastating first strike on the US, we want them to understand that this would be a stupid notion. We want them to understand that it wouldn’t even affect us by the use of our new generation technology. The purpose of a missile interceptor system is Strategic Deterrence in order to mark a beginning to the end of Intercontinental Ballistic Missiles (ICBM’s).

As one can see, this approach is to convince anyone of out there that plans to use ICBMS on the US, that it would be a bad idea. We wouldn’t only destroy their weapon before it would even intersect the coast of this country but we would counter attack, and within seconds, their little huts would be on fire. We will refer to this system as the AE3021BF11-MI. This system works in the following way, it seeks to hit missiles just before, during and right after re-entry, a few hundred miles away from the US.

The point about an ICBM is that its trajectory is ballistic, and hence very predictable. In addition, the Rocket Equation implies that to have a payload that is on the order of a few tons, the vehicle at launch must have a large gross weight. The thrust is also large, but the acceleration is still moderate, so that the large launch vehicle spends quite a bit of time moving relatively slowly in the atmosphere, while still carrying many tons of explosive fuel. It is highly vulnerable then. It is also true that a ground-launched defensive missile (one sent to intercept and destroy an incoming missile) takes a long time to climb to the speed and altitude needed for the intercept.[1]

System Elements & Mission Requirements

This section is devoted to defining the system and the 3 subsystems that compose it as well as the mission requirements and capabilities of each subsystem. The premise is that in times of tension, there will be some number of aircraft already loitering at about Mach 0.6 and maybe 40,000 feet, off the shores of the US and some over the US, giving us a height and range advantage over ground-based rocket systems.

The assumption that missile warheads will come in on a relatively shallow trajectory is important in the discussion of this paper. As the missile came down into the general area, the Theater High Altitude Air Defense (THAAD) ground-based missiles would rise very quickly to hit the incoming missile, hopefully before it broke out its packet of multiple warheads. Once the warheads separated, there was a huge issue with determining which ones were real and which ones were just duds and decoys. The warheads would simply outnumber the airplanes and air defense missiles that could be
mustered. If the warheads were also independently maneuverable, they might become much more survivable, but their CEP might also increase. The THAAD is the weak link in the chain. At least to me, it does not seem credible, except for being the only system available to respond to a sudden, surprise attack involving one or maybe a very few missiles from a well-feared irrational enemy. A rocket that must be launched from the ground, and hopes to hit something at an altitude of say 100,000 to 300,000 feet, will take quite a long time to get going and reach the target area. The available response time may be simply too short to allow this in most cases. [1]

The system elements for the previously defined system, AE3021BF11-MI, are composed of the following 3 subcomponents:

1. Transonic Patrol
2. Supersonic UCAV
3. Hypersonic Weapon

The large transonic patrol’s main purpose is to carry the other two components. It must carry lots of fuel in order to satisfy its ability to loiter for long hours in times of high stress. Inside this carrier one find extensive electronic communication and counter measure systems. It is required that this carrier can store up to four uninhabited combat air vehicles (UCAV’s) that can be launched on warning. The plan is to have multiple carriers flying at once over the periphery of the US at an efficient altitude of 40,000 ft / ~12,000 meters.

The supersonic UCAV’s purpose is to accelerate and climb up to altitudes of 150,000 ft / ~46,000 meters in order to put the hypersonic weapons, which each will carry four of them, at a reasonable altitude for hypersonic operations. These supersonic UCAV’s are expected to accelerate to speeds twice the speed of sound, in other words, Mach two. Once their mission is completed they should be able to glide their way home in order to conserve fuel and minimize overall takeoff weight of mission. An expectation holds that this supersonic vehicle will be able to reach speeds of up to Mach four.

As mentioned previously, there will be four hypersonic weapons per UCAV. They are intended to travel at hypersonic speeds using air breathing supersonic ramjets to extend the range that can be achieved by the use of rockets. Their main purpose is to quickly accelerate to any incoming Intercontinental Ballistic Missile (ICBM), and simply destroy it by direct collision or explosion. Note that recovery of the system is important, as this negates the ability of an attacker to merely threaten, or launch fakes and thus take out much of the defense system before launching the real attack. [1] In other words, we want to be able to launch and recover by far many times what any country will spend in ICBM’s. Thus, for this reason the hypersonic weapon will not be launched unless there is a real threat.
Table 1 below represents the mission requirements and description according to each stage of the mission.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Mission Requirements Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - Warning</td>
<td>Carriers will be ordered to start their engines and climb to an altitude of ~12,000 meters</td>
</tr>
<tr>
<td>1 - Signal</td>
<td>Carriers begin to climb in altitude and begin 2 minute countdown to release UCAV</td>
</tr>
<tr>
<td>2 - Release</td>
<td>UCAV is released and continues to climb while accelerating to speeds as high as 4 times speed of sound</td>
</tr>
<tr>
<td>3 - Weapon</td>
<td>ICBM trajectory is found and hypersonic weapon is released in order to bring down threat</td>
</tr>
<tr>
<td>4 - Return</td>
<td>UCAV returns home by gliding back to base while transonic patrol will be loitering in case more ICBM’s are found.</td>
</tr>
</tbody>
</table>

**TABLE 1**

Table two below represents each part of the system and characteristics in flight. Assuming the values illustrated in table to is how Akash and I were able to obtain the plots included in the paper.

<table>
<thead>
<tr>
<th>System</th>
<th>Operating Altitude (km)</th>
<th>Density (kg/m³)</th>
<th>Temperature (K)</th>
<th>Pressure (N/m²)</th>
<th>Speed (Mach #)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transonic Patrol</td>
<td>0 &lt; H &lt; 12</td>
<td>1.225 &lt; d &lt; .3108</td>
<td>288 &lt; T &lt; 216.65</td>
<td>101325 &lt; P &lt; 19330</td>
<td>0 &lt; M &lt; 0.8</td>
</tr>
<tr>
<td>UCAV</td>
<td>18 &lt; H &lt; 46</td>
<td>0.169 &lt; d &lt; 0.00164</td>
<td>216.6 &lt; T &lt; 267.85</td>
<td>7271.8 &lt; P &lt; 126</td>
<td>0.8 &lt; M &lt; 4</td>
</tr>
<tr>
<td>Hypersonic Weapon</td>
<td>18 &lt; H &lt; 56</td>
<td>0.12067 &lt; d &lt; .00003</td>
<td>216 &lt; T &lt; 204</td>
<td>7505 &lt; P &lt;1.75</td>
<td>4 &lt; M &lt; 8</td>
</tr>
</tbody>
</table>

**TABLE 2**
Hypersonic Weapon

Overall Configuration:

The overall configurations of our hypersonic weapon were obtained using some of Darpa’s own hypersonic weapon. Parameters such as length and overall width were obtained from using the previous method mentioned. A rough sketch of the hypersonic weapon is shown below. Note that this is simply a rough sketch attempting to illustrate our design without doing any calculations to make sure it met the requirements (this is done further down the paper). The sketch was made using AutoCad.

As one can see this resembles the Darpa’s hypersonic weapon..... A length of 3 meters was used for the length (this is the length of a supersonic weapon from Darpa on average) from the picture above it is easy to see that it is composed of a delta wing and a Sears Haack body. A Sears Haack body was obtained using the method shown in AE-3021. Assuming a length to diameter we were able to make calculations and obtain values such as volume of the Sears Haack body and wave drag. Some of the formulas used are shown below:

\[ s(\theta) = \frac{4V}{\pi l} (\sin\theta - \frac{\sin(3\theta)}{3}) \]

Since we know that by using pi/2 as our theta will give us our max diameter in the middle of the Sear Haack body, we were able to set this equation equal to the cross sectional area and this way we were able to solve for volume. A ratio of length to diameter of 10 was used

\[ s(\theta) = \frac{4V}{\pi l} (\sin\theta - \frac{\sin(3\theta)}{3}) = \frac{\pi}{4} d^2 \]

\[ s(\frac{\pi}{2}) = \frac{4V}{\pi^3} (\sin\frac{\pi}{2} - \frac{\sin(3\frac{\pi}{2})}{3}) = \frac{\pi}{4} (.3)^2 \]

Solving for Volume we found:

\[ V = .125 \text{m}^3 \]
A plot was made of the Sears Haack body using the previous explained method. We used MatLab in almost every single calculation and codes of our work will be provided and attached in the final project folder. From this plot one can clearly see that the length of our Sears Haack body was chosen to be three meters. We initially had chosen a length of 5 meters but then we realized this was simply a very large body which would not only increase drag but also weight. Also, since this body will be flying at really high Mach numbers, anything that comes in its way will be destroyed instantly no matter if its ten meters or 1 meter. Since this hypersonic weapon will be using ramjets as its source of propulsion instead of the usual rocket engines in order to increase its range, we chose a length that would be achievable for the ramjet since we want the fuel to light in the combustion chamber and not once it escapes into the atmosphere. A body with a length of one meter would not give the fuel enough time to light and thus its propulsion would not be efficient if it even works. A three dimensional body plot of this body was also obtain using MatLab and shown below.
Weight estimation

The weight estimation for this weapon was made by averaging various hypersonic weapons. Our results are shown in the following table shown below:

<table>
<thead>
<tr>
<th>Missile</th>
<th>Mass (Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIM-120 AMRAAM</td>
<td>152</td>
</tr>
<tr>
<td>R-77</td>
<td>175</td>
</tr>
<tr>
<td>MBDA Meteor</td>
<td>185</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>170.67</strong></td>
</tr>
</tbody>
</table>

After looking at some of the information on DARPA’s new hypersonic weapon, this weight estimate is actually pretty accurate to the weight of their weapon. The only difference between our weapon and the new hypersonic weapon, is that ours will be powered by a ramjet instead of a rocket. Having a ramjet as our propulsion system could in fact affect the weight estimate by increasing it. In this project, we have made an engineering assumption that the weight of a ramjet engine is pretty close to the rocket engine.

Sweep angle modification

For the sweep angle calculations we wanted to make sure that this body would have a subsonic tip at the nose using methods learned in AE-3021. We chose a sweep angle of 83 degrees after analyzing the body at different mach numbers. Since we know that the Mach angle \( \mu \) is equal to the \( \arcsin \) of one over Mach number, we were able to obtain a Mach angle of 7.1 by assuming that this hypersonic weapon will reach maximum Mach number of 8. It is important to remember that a subsonic tip will have a lower drag

\[
\mu = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{8}\right) = 7.18 \text{ deg}
\]

We used a sketch that was made in AutoCad to obtain the different Mach angles ranging from 4<\( M \)<8. This allowed us to make sure that the tip of this hypersonic weapon would be subsonic. What we found in this case was that our initial design did not allow us to achieve this. So we concluded that if we wanted to keep the drag of this system as low as possible we would have to increase the sweep angle to anything above 83 degrees. The picture below clearly illustrates how our initial design does not have a subsonic tip.
As a solution to our problem, it was concluded that the Sears Haack body would have to be shifted backwards in order to escape the mach angle. This would also mean that we would end up with a lower aspect ratio (by a factor of two) of the initial delta wing. Note that the Sears Haack body was only moved back and was not changed in any way.

It is clearly shown to the most careful of observers that our new body will now be able to have a subsonic tip by making the sweep angle 83 degrees. This of course is only true by making sure this weapon never reaches Mach numbers higher than 8. A picture below allows the reader to see the changes made to the new weapon.
Analysis:

Before proceeding with the analysis of our hypersonic weapon, we want to talk a little about the assumptions we made. From the paper given in AE-3021 we see that one of the requirements for the hypersonic weapon is to be able to fly at conditions ranging from altitudes of 60,000ft to 250,000ft which translates to SI units as 18km to 53km. Using tables found in Andersons book and online, we were able to find some of the characteristics of such altitudes. Since our lift and drag are functions of the atmospheric conditions that this weapon is traveling through, we decided to choose four altitudes where one could interpolate if interested in a value in between. The following table captures the characteristics of these chosen altitudes where performance parameters were obtained.

<table>
<thead>
<tr>
<th>Altitude (Km)</th>
<th>Density (kg/m^3)</th>
<th>Temperature (K)</th>
<th>Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>0.12067</td>
<td>216.65</td>
<td>7505</td>
</tr>
<tr>
<td>38</td>
<td>0.00518</td>
<td>245.45</td>
<td>365.45</td>
</tr>
<tr>
<td>53</td>
<td>0.000682</td>
<td>265</td>
<td>51.86</td>
</tr>
<tr>
<td>76</td>
<td>0.00003</td>
<td>204</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Table 4

Lift:

The lift for this hypersonic weapon was obtained using the properties of the standard atmosphere shown on table 4. By using the Newtonian method, we were able to solve for the normal force caused by the flow and solved for the lift force by defining \( L = N \cos \alpha \). We were able to find \( N \) using the following method. First we took Mach number ranging from four to eight. Then we took the four different altitudes and solved for the speed of sound using the temperature variations. Since we knew that the area of the delta wing is equal to 1.038m^2, we took the density from the corresponding altitudes and used the Newtonian formula. We also used a variation of angles of attack in order to obtain different lifts at different angles of attack.

\[
N = \rho U^2 A \sin^2(\alpha) \quad L = N \cos \alpha
\]
From the previous plots we were able to make the following observations:

Lift decreases with altitude substantially. This is due to the fact that density is so low at such high altitudes that. We see our lift force decrease all the way down from 10,000N to only 100N ranging from altitudes of $18\text{km} < H < 76\text{km}$. 
Drag:

Wave drag for the Sears Haack body was obtained using the methods shown in AE-3021 in the Sears Haack body section. This wave drag of course does not account for the delta wing and is only the wave drag of the body. The plots below were obtained by using the previous volume found, which we found the volume to be .125 m$^3$. Density variations accounted for as well as temperature variations using the values given in chart 4. The following plots, as one would expect, represent the wave drag increasing as we go faster and faster. This is of course common sense. Using the following formula we were able to plot the values for wave drag for different altitudes and speeds.

\[ D_w = \frac{64 \rho \omega U \omega V^2}{\kappa l^4} \]

Wave drag Coefficient of Sears Haack body of the hypersonic weapon was found also using the slide show from AE-3021 and calculated below.

\[ C_{Dw} = \frac{24V}{l^3} = \frac{24 \cdot 125}{3^3} = .333 \]
The Drag force for the hypersonic weapon was obtained using the Newtonian methods found in Anderson’s fundamental of aerodynamics. By calculating the normal force as explained above in the lift section, we were able to calculate drag using $D = N \sin \alpha$.

$$N = \rho U^2 A \sin^2(\alpha)$$

$$D = N \sin \alpha$$

It is obvious from the previous plots that drag increases as we go up in Mach number. Also, from the previous plots the observer can see that the Drag force decreases as we go up in altitude. This is due to the fact that the density of the air is decreasing as we go up in altitude, thus, the $N$ vector decreases as a whole. However, it is important to note that the Lift force will decrease as well. One should see that there is no free lunch.
L/D, Cl, Cd

We were able to obtain a plot of the lift to drag of our hypersonic weapon using the exact shock expansion theory since it is more accurate than the Newtonian method. A constant Mach number of 6 was used.

As we can see from the plot below, lift to drag decreases with increasing the angle of attack of our weapon. This makes sense since we will have more drag on the a bigger cross sectional area that the flow will see as angle of attack increases. We decided to include. It is interesting to see that both the lift coefficient and the drag coefficient increase with angle of attack. Lift coefficient remains higher than our drag coefficient which in turn tells us that our lift to drag is a value greater than one.

Aerodynamic properties of a flat plate based on Newtonian theory
Range and Endurance:

In order to obtain a good range and endurance, one greater than a rocket engine, there is a requirement to use a ramjet as our propulsion system. This configuration will allow both of these performance parameters to increase substantially. This is something important if we consider that our hypersonic weapon will be chasing ICBM’s.

A range was estimated using “Roadmap for the Hypersonics Programs of the Department of Defense” and thermodynamics of propulsion. Note that this range was calculated assuming this object is traveling under atmospheric conditions at 56km and Mach number of 8.

<table>
<thead>
<tr>
<th></th>
<th>Length (in)</th>
<th>Launch Weight (lb)</th>
<th>Cruise Speed / Altitude (ft)</th>
<th>Range (nm) / Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supersonic Med-Range</td>
<td>168 to 218</td>
<td>2,200 to 3,000</td>
<td>Mach 4 / 80,000</td>
<td>250 / 7 and 500 / 13</td>
</tr>
<tr>
<td>Supersonic Long-Range</td>
<td>154</td>
<td>1,833</td>
<td>Mach 3.5 / 80,000</td>
<td>750 / 21 and 1000 / 29</td>
</tr>
<tr>
<td>Hypersonic Med-Range</td>
<td>168</td>
<td>2,200</td>
<td>Mach 7 / 95,000</td>
<td>250 / 5 and 500 / 9</td>
</tr>
<tr>
<td>Hypersonic Long-Range</td>
<td>168 to 300</td>
<td>2,800 to 5,000</td>
<td>Mach 7 / 95,000</td>
<td>750 / 12 and 1000 / 16</td>
</tr>
</tbody>
</table>
UCAV

OVERALL CONFIGURATION:

The overall configurations of our Unmanned Combat Air Vehicle were obtained using some of the UCAV’s currently used in the US Navy. Parameters such as length and overall width were obtained from using the previous method mentioned. A rough sketch of the hypersonic weapon is shown below. Note that this is simply a rough sketch attempting to illustrate our design without doing any calculations to make sure it met the requirements (this is done further down the paper). The sketch was made using AutoCad.

It is easy to see the resemblance of the Navy’s UCAV to the AE-3021 UCAV (It is indeed a derivation of the Navy’s UCAV). With a length of 9 meters in length and 4.34 meters in width this bad boy is able to cruise from compressible subsonic speeds to outstanding velocities of up to four times the speed of sound. It is easy to see that it is composed of a delta wing and a Sears Haack body. A Sears Haack body was obtained using the method shown in AE-3021. Assuming a length to diameter of 10 we were able to make calculations and obtain values such as volume of the Sears Haack body and wave drag. Some of the formulas used are shown below:

\[ s(\theta) = \frac{4V}{\pi l} \left( \sin \theta - \frac{\sin(3\theta)}{3} \right) \]

Since we know that by using pi/2 as our theta will give us our max diameter in the middle of the Sears Haack body, we were able to set this equation equal to the cross sectional area and this way we were able to solve for volume. A ratio of length to diameter of 10 was used

\[ s(\frac{\pi}{2}) = \frac{4V}{\pi l} \left( \sin \frac{\pi}{2} - \frac{\sin(3\frac{\pi}{2})}{3} \right) = \frac{\pi}{4} d^2 \]

Solving for Volume we found:

\[ V = 3.372m^3 \]
A plot was made of the Sears Haack body using the previous explained method. From this plot one can clearly see that the length of our Sears Haack body was chosen to be three meters. We initially had chosen a length of 9 meters but then we realized this was simply a very large body which would not only increase drag but also weight. Also, since this body will be flying at really high Mach numbers, anything that comes in its way will be destroyed instantly no matter if its ten meters or 1 meter. Since this hypersonic weapon will be using ramjets as its source of propulsion instead of the usual rocket engines in order to increase its range, we chose a length that would be achievable for the ramjet since we want the fuel to light in the combustion chamber and not once it escapes into the atmosphere. A body with a length of one meter would not give the fuel enough time to light and thus its propulsion would not be efficient if it even works. A three dimensional body plot of this body was also obtain using MatLab and shown below.
Wave drag of the Sears Haack body of the UCAV was found and shown in the following plots. This wave drag of course does not account for the delta wing and is only the wave drag of the body. The plots below were obtained by using the previous volume found, which we found the volume to be 3.372m³. Density variations accounted for as well as temperature variations using the values given in table 5. The plots as one would expect, represent the wave drag increasing as we go faster and faster. This is of course common sense. Using the following formula we were able to plot the values for wave drag for different altitudes and speeds.

\[
D_w = \frac{64\rho_\infty \alpha^2 V^2}{\kappa l^4}
\]

Wave drag coefficient for the Sears Haack body of the UCAV was also found using the notes given in AE-3021 as shown previously in the hypersonic weapon section.

\[
C_{DW} = 24 \frac{V}{l^3} = 24 \frac{3.372}{9^3} = .111
\]
Sweep angle was calculating by taking the highest Mach number that the UCAV will obtain while maintaining its tip subsonic. Since our UCAV will go up to Mach 4, we must give it a sweep angle of 77 degrees in order to have a major drag reduction.

\[ \mu = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{4}\right) = 14.5 \text{deg} \]

The picture below is a rough sketch of the UCAV. In this top view of the system, one is able to see how the Mach angles change with Mach number. A sweep of 77 degrees was used in order to ensure good performance.

In the picture above we have tried to illustrate how this system would work with the four hypersonic weapons it's supposed to carry. After different configurations were discussed, it was concluded that this would be the most efficient way to carry they hypersonic weapons since it would allow us to build the UCAV at the desired sweep angle. It is assumed that the hypersonic weapons will be attached to the bottom of the UCAV. A ram jet will be providing thrust to the UCAV since it is expected to cruise at critically high Mach numbers.
Weight Estimation:

The weight estimation for the UCAV was done in the following way. First we calculated the payload, which turned out to be four times the weight of each hypersonic weapon. Then we looked at different UCAV’S with about the same payload weight and took a payload to takeoff weight ratio and averaged the ratio obtained. Once this was done we multiplied our payload with the average ratio to get the takeoff weight of our UCAV. The table below summarizes the previously explained method.

<table>
<thead>
<tr>
<th>Hypersonic Weapon Weight (Payload of UCAV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Missile</td>
</tr>
<tr>
<td>AIM-120 AMRAAM</td>
</tr>
<tr>
<td>R-77</td>
</tr>
<tr>
<td>MBDA Meteor</td>
</tr>
<tr>
<td>Average (Each)</td>
</tr>
<tr>
<td>Total Payload of UCAV</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Payload Mass</th>
<th>Wt/o(Kg)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>MQ-1 Predator</td>
<td>284.4</td>
<td>1020</td>
<td>0.28</td>
</tr>
<tr>
<td>MQ-9 Reaper</td>
<td>1700</td>
<td>4760</td>
<td>0.36</td>
</tr>
<tr>
<td>X-47B</td>
<td>2000</td>
<td>20215</td>
<td>0.1</td>
</tr>
<tr>
<td>EURO HAWK</td>
<td>1360</td>
<td>14628</td>
<td>0.09</td>
</tr>
<tr>
<td>Gray Eagle</td>
<td>448</td>
<td>1451</td>
<td>0.31</td>
</tr>
<tr>
<td>Average</td>
<td>5792.4</td>
<td>42074</td>
<td>0.14</td>
</tr>
<tr>
<td>Our Aircraft</td>
<td>682.68</td>
<td>871.43</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Analysis:

From the paper given in AE-3021 we see that one of the requirements for the UCAV is to be able to fly at conditions ranging from altitudes of 18km to 46km. Since our lift and drag are functions of the atmospheric conditions that this weapon is traveling through, we decided to choose four altitudes where one could interpolate if interested in a value in between. The following table captures the characteristics of these chosen altitudes where performance parameters were obtained.

<table>
<thead>
<tr>
<th>Altitude (Km)</th>
<th>Density (kg/m^3)</th>
<th>Temperature (K)</th>
<th>Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>0.169</td>
<td>216.6</td>
<td>7271.8</td>
</tr>
<tr>
<td>27</td>
<td>0.0288</td>
<td>223.65</td>
<td>1847.5</td>
</tr>
<tr>
<td>35</td>
<td>0.0082</td>
<td>237</td>
<td>559</td>
</tr>
<tr>
<td>46</td>
<td>0.00164</td>
<td>267.85</td>
<td>126</td>
</tr>
</tbody>
</table>

TABLE 5
Supersonic Lift

\[ L = \frac{b}{2} \int \frac{\partial L}{\partial \alpha} \, d\gamma = \frac{\pi}{4} \rho_\infty U_\infty^2 b^2 c \]

The following plots are supersonic values for the lift force. They were obtained using the notes found in AE-3021 under wings and bodies in compressible flow. As we can see from the plots, the lift force decreases as we go up in altitude. This is of course expected since density begins to decrease as we go up in altitude. The following plots also include changes in angle of attack. Since we assumed that our wings are composed of a thin, uncambered delta wing, we see that we do not produce lift at zero angle of attack.
Supersonic Lift coefficient

The following plot represents values for the lift coefficient in supersonic flight of our UCAV. They were obtained using the notes found in AE-3021 under wings and bodies in compressible flow. From the following plot the reader should be able to see that lift coefficient is greater at higher angles of attack. This is obvious to anyone with an aerospace background. From the formulas below we also see that the lift coefficient does not depend on altitude in any way thus only one plot was generated for this section.

\[
C_l = \frac{2}{\sqrt{M^2}} \int_0^1 (\alpha + \alpha) d\left(\frac{X}{c}\right)
\]

\[
C_l = \frac{4\alpha}{\sqrt{M^2} - 1}
\]
Supersonic Drag coefficient

The following plot represents values for the drag coefficient in supersonic flight of our UCAV. They were obtained using the notes found in AE-3021 under wings and bodies in compressible flow. From the following plot the reader should be able to see that drag coefficient is greater at higher angles of attack. This is obvious to anyone with an aerospace background. From the formulas below we also see that the drag coefficient does not depend on altitude in any way thus only one plot was generated for this section. It is also easy to see after comparing to the plot showing the lift coefficient that the values for the drag coefficient are lower for a given Mach number and angle of attack. This is good since we want our lift to drag to be as high as possible. The following plot was made using the following formulas learned in class.

\[ C_d = \frac{4}{\sqrt{M_\infty^2 - 1}} \int \left[ \theta_t^2 + \theta_c^2 + \alpha^2 \right] d\left( \frac{x}{c} \right) + \left( C_{d, friction} \right) \]
Coefficient of friction – The reference temperature method

Assuming that turbulent flow begins to act on the body from the chord tip to the chord end, Akash and I were able to calculate for the coefficient of friction using the reference temperature approach. First we had to make sure that our Reynolds number was on the magnitude of Re> 10^6 in order to use the Boeing correlation for reference temperature. As an assumption, it was assumed that the wall was adiabatic, meaning that Tw = T*. Once this was done, We proceeded by using the Cf formula learned in AE-3021 shown below:

\[
\frac{T^*}{T_1} = 1 + 0.1198M_1^2
\]

\[
Cf = 0.295 \frac{T^*}{T} \left[ \log \left( Re \times \frac{T}{T^*} \frac{\mu}{\mu^*} \right) \right]^{2.45}
\]

\[
Re = \frac{\rho V \infty c}{\mu} > 100,000
\]

Note that in order to find \(\mu\) which was used for Re #, we assumed a sea level viscosity of air of 1.8E-5
Turn radius plots at different altitudes and Mach numbers

Using Anderson’s Aircraft performance and design book we found that the minimum turn radius is expressed in the formula shown in the right. From this equation we are able to see that the turn radius depends only on velocity and load factor \( n \). Since this is an unmanned air combat vehicle, it is allowed to sustain greater g forces. Having a pilot in an aircraft constrains the load factor to about 8, anything greater than that and the pilot would become unconscious. Thus, a load factor of 15 was assumed for this vehicle. From the formula given above we see that to obtain the smallest possible radius, two things must happen.

1. We want the highest possible load factor (highest possible L/W)
2. the lowest possible velocity
From the previous plots it is easy to see that the turn radius decreases as we go up in altitude. This is due to the fact that temperature decreases as we go up in altitude and we know from the definition of Mach number that Mach number is inversely proportional to the square root of temperature.
Turn rate at different altitudes and Mach numbers

Using Anderson’s Aircraft performance and design book we found that the turn rate of our UCAV could be expressed in the formula shown in the right. We see that to obtain the largest possible turn rate we want the highest possible load factor and the lowest possible velocity. These conditions are clearly the same as for the smallest possible Radius.

\[ \omega = \frac{g_0 \sqrt{n^2 - 1}}{V_\infty} \]
Turn Rate vs. Mach No. at 35Km

Turn Rate (rad/sec)

Mach No.

Turn Rate vs. Mach No. at 46Km

Turn Rate (rad/sec)

Mach No.
Service ceiling at full load and Rate of climb:

The picture below is a representation of our service ceiling at a full payload and from here we see that it occurs at an altitude of 27km. This is good because it fits as one of our altitude options. (you were right!!) From the graph below we were able to calculate the maximum rate of climb also and an absolute ceiling. The red dot represents our maximum ceiling which is really not useful but good to know parameter. The blue graph below represents the maximum rate of climb as it begins to decrease with altitude. As we see, rate of climb varies pretty linearly with altitude. The green graph simply represents where the rate of climb would be equal to zero.
Range:

The graph below is a representation of our range versus Mach number at different altitudes. Obviously our range will be greater for a higher altitude due to the fact that density is lower and thus drag is smaller. Focusing on the red graph which is the range at 46 km we see that it keeps increasing. This is due because each range graph follows a pattern and we are not able to see where it reaches its maximum. At lower altitudes it is easy to see that our range is really low from the great amounts of drag due to flying at such high velocities in such a dense atmosphere. If we were to cruise at an average 35km attitude going Mach 2.7 we could expect to see values of range as high as 20 km. This value may see relatively high but the reader should take into account that our UCAV is traveling at speeds that are close to 3 times the speed of sound in this case.
Endurance:

From the plot below we were able to capture how endurance changes with Mach number at various altitudes. Even though this is an unusual plot, it seems to show some pretty cool data. From this plot we are able to see that in order to obtain the best possible endurance we must either fly at an altitude of 35 km going Mach 2.5 or fly at an altitude of 45 km going Mach 4. This would give us a maximum endurance of 7.5 hours which is more than necessary to complete the mission and turn around and glide back home. Other configurations are possible like going Mach 1.3 at an altitude of 27 km, though we want to get the highest possible Mach number since we will literally be chasing ICBM’s.
Transonic Patrol

As explained in the introduction the transonic patrol is in charge of loitering around the perimeter of the US in times of high stress. This transonic patrol must be able to carry the weight of 4 UCAV’S talked about previously which in turn will carry four hypersonic weapons each. Thus, this transonic patrol must be as efficient as possible and process characteristics of a carrier. Also, by definition, a transonic patrol will be operating at transonic speeds.

The table below illustrates the flight characteristics of this transonic patrol.

<table>
<thead>
<tr>
<th>Operating Altitude (Km)</th>
<th>Density (Kg/m³)</th>
<th>Temperature (K)</th>
<th>Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.225</td>
<td>288</td>
<td>101325</td>
</tr>
<tr>
<td>6</td>
<td>0.6597</td>
<td>249.15</td>
<td>47181</td>
</tr>
<tr>
<td>12</td>
<td>0.3108</td>
<td>216.65</td>
<td>19330.4</td>
</tr>
</tbody>
</table>

The overall configuration of the carrier was obtained using the method explain in AE 3021 by Dr. Komarath were we make multiple iterations until the value of range and endurance wanted is obtained. Once we were able to obtain the range and endurance wanted we were able to play with performance parameters and used these to improve our results. Some of the performance parameters found are shown below on table ___

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value obtained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing Span (m)</td>
<td>60.96</td>
</tr>
<tr>
<td>Wing Area (m^2)</td>
<td>160.8</td>
</tr>
<tr>
<td>Wing Loading (N/m^2)</td>
<td>4,421.15</td>
</tr>
<tr>
<td>L/D (Assuming steady level flight)</td>
<td>17.8</td>
</tr>
<tr>
<td>T/W</td>
<td>0.22</td>
</tr>
<tr>
<td>AR</td>
<td>8.69</td>
</tr>
</tbody>
</table>
Weight estimation

Weight estimation was made by averaging a few of the aircraft mentioned previously. Iterations were made to the process of choosing a wing area and other important parameters until we were able to obtain the endurance wanted. The following chart summarizes the process for our weight estimation.

This method of finding the weight of the carrier is estimated by acutely finding values for each and every single part that forms a carrier and then iterating this process to get the most efficient values for lift to drag, range and endurance.

To kick off this process, we first found the payload mass as following:

<table>
<thead>
<tr>
<th>Payload Weight</th>
<th>Type</th>
<th>Quantity</th>
<th>Per Unit (Kg)</th>
<th>Mass(Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCAV</td>
<td>4</td>
<td>4,275.99</td>
<td></td>
<td>17,103.96</td>
</tr>
<tr>
<td>Weapons</td>
<td>16</td>
<td>170.67</td>
<td></td>
<td>2,730.67</td>
</tr>
<tr>
<td>Total Payload(Kg)</td>
<td></td>
<td></td>
<td></td>
<td>19,834.62</td>
</tr>
<tr>
<td>Total Payload Weight(N)</td>
<td></td>
<td></td>
<td></td>
<td>194,379.30</td>
</tr>
</tbody>
</table>

Then we looked the crew required for a carrier of such status and decided to equip it with a pilot and a co-pilot and an avionics engineer to provide help incase of any emergency.

<table>
<thead>
<tr>
<th>Crew Weight</th>
<th>Type</th>
<th>Quantity</th>
<th>Per Unit (Kg)</th>
<th>Mass(Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pilot</td>
<td>2</td>
<td>95</td>
<td></td>
<td>190</td>
</tr>
<tr>
<td>Avionics Engineer</td>
<td>1</td>
<td>95</td>
<td></td>
<td>95</td>
</tr>
<tr>
<td>Total (kg)</td>
<td></td>
<td></td>
<td></td>
<td>285</td>
</tr>
<tr>
<td>Total Weight(N)</td>
<td></td>
<td></td>
<td></td>
<td>2793</td>
</tr>
</tbody>
</table>

After calculating the weight of the payload and the weight of the crew riding the carrier, we looked at multiple different carriers that contain the same payload to determine our engines. After considering many different engines, we finally decided to go with General Electric GE90 as it provides the thrust we need for a relatively low weight.

<table>
<thead>
<tr>
<th>Engine</th>
<th>Type</th>
<th>Mass(Kg)</th>
<th>Max Thrust(KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Electric GE90</td>
<td>Turbofan</td>
<td>3775</td>
<td>416.8</td>
</tr>
</tbody>
</table>
The Electrical and the Avionics requirements the presented in the problem statement were meticulously researched over the Internet and its weight was estimated to be around 1929Kg. Then we looked at multiple planes that have carried similar payload weight to our carrier and averaged their structural weight. The on-board avionics, the structural weight combined with the weight of the engines gives the empty weight of the carrier.

<table>
<thead>
<tr>
<th>Empty Weight</th>
<th>Type</th>
<th>Quantity</th>
<th>Per Unit (Kg)</th>
<th>Mass (Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronics</td>
<td>1</td>
<td>1929</td>
<td></td>
<td>1929</td>
</tr>
<tr>
<td>Engine</td>
<td>4</td>
<td>3775</td>
<td></td>
<td>15100</td>
</tr>
<tr>
<td>Structure</td>
<td></td>
<td></td>
<td></td>
<td>108825</td>
</tr>
</tbody>
</table>

Total Empty Mass (Kg) 110754
Total Empty Weight (N) 1085389.2

The empty weight of the plane, the payload weight of the fuel and the TSFC of the engine were used to iterate for the fuel needed to get a reasonable endurance. Along with this recapitulation, we looked at airplanes with similar requirements and decided a fuel weight of 10,000 gallons would suffice the problem statement.

<table>
<thead>
<tr>
<th>Fuel Weight</th>
<th>Type of fuel:</th>
<th>JP-4 JET FUEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (kg/gal)</td>
<td>3.10</td>
<td></td>
</tr>
<tr>
<td># of gallons needed</td>
<td>10,000.00</td>
<td></td>
</tr>
<tr>
<td>Total fuel weight (N)</td>
<td>304,404.30</td>
<td></td>
</tr>
<tr>
<td>Total fuel Mass (Kg)</td>
<td>31,061.66</td>
<td></td>
</tr>
</tbody>
</table>

By adding all the weights calculated above gave us the following final takeoff weight:

| Total Takeoff Weight (kg) | 161,935.29 |
| Total Takeoff Weight (N) | 1,586,965.80 |
| Total Empty Weight (kg) | 130,873.62 |
| Total Empty Weight (N) | 1,282,561.50 |
The diagram below is a representation of the process explained above.
Implications for takeoff

We are pretty confident that this transonic patrol will have combined features of the following aircraft:

1. B-52
2. KC-135
3. C-5
4. C-130

By assuming that the diameter of the fuselage of our transonic patrol aircraft is large enough to carry the supersonic UCAV’s inside, which is a good assumption, we are able to come up with a configuration that will enable and facilitate the implications for takeoff.

As we have seen before, the span of the delta wing of the UCAV is about 4.3m in length. By averaging the fuselage diameters we found that this value is in fact a very close value to the value of the diameter of our transonic patrols fuselage diameter. The UCAV’s would be located one after the other, in series.

Thus, the implications for takeoff are simple. The goal is to simply load the carrier through the back door. Obviously this would mean we need to create some sort of configuration that will in fact not only hold the UCAV’s in place but also be able to eject them upon order. After this is done, takeoff would simply mean getting the carrier off the ground and no black magic attached. We do not need to hang the UCAV’s from the wings or none of that nonsense. We only need to load the carrier properly and start our engines. Having this said, the implications for takeoff are the same as for any aircraft.
Critical Mach Number

Calculating critical Mach number for our aircraft was pretty simple using the method learned in AE-3021. By solving for $C_p$ min and converting to a compressible correction, we calculated the critical Mach number with the equation shown below.

\[
c_{p} = \frac{2}{\gamma M_{\infty}^2} \left[ \left( \frac{\gamma + 1}{2} \right) \left( \frac{\gamma - 1}{2} M_{\infty}^2 \right)^{1-\gamma} - 1 \right]
\]

Under the assumption that our airfoil is a NACA 0012, we were able to obtain data from Anderson’s aerodynamics book. We found that the $C_p$ min for low speed at zero angle of attack was equal to $C_p$ min = .43. Thus, by making a compressibility correction we were able to find $C_p$ min and were able to solve for $M$ crit from the equation above.

\[
C_{p_{min}} = \frac{C_{p_{min0}}}{\sqrt{1 - M_{crit}^2}}
\]

By assuming that the ratio of specific heats is equal to 1.4, we found the critical mach number of our transonic patrol equal to .74.

\[
M_{crit} = .74
\]

Note that this value was obtained under the following assumptions:

1. Ratio of specific heats of 1.4
2. Zero angle of attack and airfoil chose to be an NACA 0012
L/D:

A plot of the lift to drag of our aircraft was obtained using the following method: We simply solved for the lift coefficient of our aircraft as well as the drag coefficient and we knew that lift to drag is equal to cl over cd.

From this plot we are able to see that our maximum lift to drag of the transonic patrol will vary with altitude. We see that our maximum lift to drag is equal to 18. The cool thing here is that this value of 18 occurs at Mach number of only .3 for sea level conditions and begins to decrease after that. This tells us that we need to climb to a higher altitude in order to get our maximum lift to drag at a reasonable speed. The Blue line signifies our lift to drag at an altitude of 6km. From this line we are able to see that at this altitude we are able to obtain our maximum lift to drag at Mach number .5. Thus we see that this value agrees with our theory that the higher altitude we achieve, the better the lift to drag at a higher mach number. Lastly, from the green line we are able to see that our maximum lift to drag of 18 occurs at Mach .7 for an altitude of 12 km. This is good since this transonic aircraft will be flying at Mach number less than .74 since this is our critical mach number. If we exceed this speed, our carrier will experience lots of drag and our lift to drag will be decreasing substantially.
Range:

Range for our aircraft was calculated using Breguet's formula for range. As we can see from the plot below we are able to obtain different ranges for different altitudes at different speeds. Since our aircraft will be flying at an average of 12km, one should put their focus on the red line. From this line we are able to see that we can obtain a range of 11,000 Km for cruise at Mach .75. Again it is important to note that this is only true for an altitude of 12km. If we go down in altitude to 6km, we can obtain a somewhat similar value for the same speed. The only difference is that this would be our maximum value as opposed to 12km where we can increase our range by increasing our Mach number. So from this plot we are able to see very interesting information. Obviously, at sea level flight our maximum range of 7,000km happens at Mach of only .46.

Again, this tells us that our transonic patrol is more efficient at higher altitudes as theory would agree with us since density and our lift over drag are going to be smaller and greater correspondingly.
Radius / Area covered and Area covered after two minute warning

From the charts above we were able to calculate the range of this aircraft. From this range of 11 km at Mach 0.8, we were able to calculate an estimate of the area that this transonic patrol will be able to cover. From the picture below, it is easy to see that this aircraft will be able to loiter at an area that is substantially big. This is good because we will have more than one aircraft monitoring around the coast line and beyond at the same time.

As a requirement stated in the paper, we calculated an average area that the transonic patrol would cover given a two minute warning to launch the UCAV’s. In this two minute period, the aircraft is assumed to be going the opposite way and turn around and follow the ICBM. We were able to calculate an estimate of the area that this carrier would cover by integrating the velocity of the transonic patrol with respect to time from zero to 120s which is the two minute warning. The picture below is a graphical representation of our results and proves why it is important to have more than one transonic patrol at the same time.
Endurance:

As we can see from the plot below us, we found the endurance for this transonic patrol as a function of Mach number and altitude. Since we will be flying this transonic patrol it is important to set our focus to the red graph. This red graph indicates endurance at an altitude of 12 km. However, it is important to see how the performance would change as we change our flying altitude. Obviously our endurance will be greater at higher altitudes since there is less drag from the density being lower. We found a maximum endurance of 14 hours at an altitude of 12km if we travel at Mach number = 0.8. We see the same endurance if we travel at an altitude of 6km traveling at an Mach number of 0.55. Thus we are able to see how our endurance changes with mach number and altitude from this plot.

Thus, Akash and I concluded that for a maximum endurance we need to fly at Mach 0.55 - 0.6 if we are going to be traveling at an altitude of 6km. This makes a lot of sense and is a value that is reasonable; it is basically what we were expecting to obtain in the end.
References

1) Class Notes

2) Missile Interceptor. Draft v0.0.

