Critical Mach Number

Consider the following situation

\[ \mathbf{M} < 1 \]

\[ U_\infty \]

\[ x/c \]

\[ T_0 \]

\[ T \]

\[ \mathbf{M} \]

\[ \mathbf{T} \]

\[ \mathbf{M}_\infty \]

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As subsonic air flow over an airfoil (or wing), it accelerates, reaches a maximum speed and then decelerates toward the trailing edge.

Since $T_0 = \text{constant}$,

$$T = T_0 - \frac{u^2}{2c_p} = T_0 - \frac{u^2 r - 1}{2R\gamma}$$

Thus, the Mach number of the flow increases and then decreases. The magnitude of this change depends on the airfoil shape and the angle of attack. Thus, it is evident that, as you increase $M_{\infty}$, the highest local $M$ on the surface may exceed 1 long before $M_{\infty}$ reaches 1.

The value of $M_{\infty}$ at which the highest $M$ on the airfoil first reaches 1 is called the critical Mach number $M_{\alpha}$.

Note

1. $M_{\alpha}$ is a value of the freestream Mach number.
2. $M_{\alpha}$ is less than 1 for anything with thickness at all.
Calculation of $M_x$

We know that

\[
\frac{P}{P_0} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{1-\gamma}}
\]

(Isentropic flow relation)

\[
\frac{P_\infty}{P_0} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2 \right)^{\frac{\gamma}{1-\gamma}}
\]

\[
\frac{P - P_\infty}{\frac{\gamma}{2} M_\infty^2 P_\infty} \equiv c_p = \frac{2}{\gamma M_\infty^2} \left( \frac{P}{P_\infty} - 1 \right)
\]
or,

\[ c_p = \frac{2}{\gamma M^2_{\infty}} \left[ \left( \frac{\gamma - 1}{2} M^2_{\infty} \right)^{1-\gamma} - 1 \right] \]

At \( M_{\infty} = M_{\infty_r} \), \( M \) reaches 1 somewhere. The value of \( c_p \) at this point can be found by setting \( M=1 \).

\[ c_p = \frac{2}{\gamma M^2_{\infty}} \left[ \left( \frac{\gamma + 1}{2} \right)^{1-\gamma} - 1 \right] \]
Note:

1. For any given airfoil, the value of \( M_{\alpha_c} \) can be found from the value of the minimum \( c_p \) on the airfoil.

2. If you have only low-speed data on the airfoil, the \( c_p \) values can be converted to values at a given subsonic Mach number using the Prandtl-Glauert relations.
Airfoils in Transonic Flow

What happens when $M_v > M_{cr}$?

subsonic

normal shock

supersonic region

subsonic flow
The flow accelerates over the front part of the airfoil. Before the point of minimum pressure is reached, it goes supersonic. Once the point of minimum pressure is passed, the flow experiences an adverse pressure gradient (pressure increases downstream). Several things happen.

The boundary layer begins to get thicker. Note that information can move upstream through the boundary layer, because the velocity is subsonic (has to reach zero at the wall).

The flow is forced to turn because of the thickening boundary layer. Compression waves are formed. These merge into one (or more) oblique shocks (or a normal shock depending on the Mach number and surface curvature, Reynolds number of the boundary layer etc.)
The pressure rises suddenly across the shock(s). The boundary layer thickens much more. It may separate!

The supersonic region ends in a normal shock and the flow becomes subsonic and decelerates. Drag increases greatly because of the shocks. The wing may stall because of the boundary layer separation.
Supercritical Airfoils

The preceding discussion shows that if \( M_{\infty} > M_c \), drag rises greatly. How can \( M_c \) be increased?

1. Use a very thin airfoil with a sharp leading edge. This is impractical for airliners (where would fuel be stored?).
2. Reduce the curvature on the upper surface. This reduces the acceleration and deceleration of the flow, so that any shocks formed will be relatively weak.

Such airfoils were among the first to be designed using detailed mathematical computation.
Comparison of transonic flow over a "usual" (NACA 64A series) airfoil with transonic flow over a supercritical airfoil.

From Bertin & Smith, 2nd Edition, p. 366

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Transonic Drag Rise

We have seen that if $M_{\infty} > M_{crit}$, shocks form above the airfoil. This causes a large increase in drag.
In addition, the expressions for the supersonic wave drag coefficient have $\sqrt{M^2_\infty - 1}$ in the denominator. Thus, the supersonic wave drag coefficient also becomes very high in the transonic range. Some people believe that linear theory could be close to the truth at $M \approx 1$, so that $C_d \to \infty$. Thus, they hypothesized the existence of the "sound barrier". In practice, $C_d$ does become quite high at $M_\infty = 1$, but fortunately stays finite, so that powerful engines can still accelerate the aircraft through $M_\infty = 1$ without needing infinite thrust.

Conclusions:

1. Good aerodynamic design dictates that the surface slopes should be smooth and gentle to avoid sharp peaks in the pressure distribution.
2. The value of $M_{cr}$ must be kept high for transonic flight.
3. Flight at $M \approx 1.0$ should be kept to a short duration.
Variation of $C_l$ with $M_\infty$
Sweep

Obviously, it is desirable to reduce the Mach number of the flow over the airfoil section. A long time ago, it was discovered that the flow could be "fooled" by simply yawing the wing.

It was discovered that the characteristics of the yawed wing at $\Omega = M_{\infty}$ were similar to those of the straight wing at $M = M_{\infty} \cos \Omega$.
Consider the flying-wing shown. If $M_{\infty} > 1$, the compression waves from the apex will be felt within the angle $\mu$ (mach angle). However, in the figure, every point on the leading edge of the wing is in the undisturbed supersonic flow, and cannot feel the compression and flow deceleration due to the other points. This is a wing with a "supersonic leading edge".
Now consider the arrow-wing shown below. Here, every point on the leading edge is within the region of disturbed (decelerated) flow caused by the apex. As a result, the Mach number at the leading edge is slightly smaller than the free-stream Mach number. This is a "subsonic leading edge". Note that the flow is still supersonic.
**Transonic Area Rule**

Within the limitations of small perturbation theory, at a given transonic Mach number, aircraft with the same longitudinal distribution of cross-sectional area, including fuselage, wings and all appendages will, at zero lift, have the same wave drag.

**Why:** Mach waves under transonic conditions are perpendicular to flow.

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Implication:

Keep area distribution smooth, constant if possible. Else, strong shocks and hence drag result.

Wing-body interaction leading to shock formation:

Observed: $c_p$ distributions are such that maximum velocity is reached far aft at root and far forward at tip.

Hence, streamlines curves in at the root, compress, shock propagates out.